Why HANK Matters for Stabilization Policy

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Abstract

When do optimal inflation and quantities differ significantly between Heterogeneous-Agent (HA) and Representative-Agent (RA) models, and what are the underlying mechanisms? To answer this question, we derive jointly the optimal fiscal-monetary Ramsey policy in HA and RA models that incorporate both price and wage stickiness. We examine different sets of fiscal tools and analyze both supply and demand shocks. Our findings show that HA economies diverge significantly from RA economies when the severity of credit constraints varies over time, which is the case for demand shocks but less so for supply shocks. Furthermore, inflation dynamics differ between HA and RA economies in response to demand shocks, particularly when fiscal policy is not employed as a stabilization tool over the business cycle. We identify the relevant fiscal tools to reduce inflation volatility over the business cycle.

Keywords: Heterogeneous agents, wage-price spiral, inflation, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

1 Introduction

Heterogeneous-Agent (HA) models describe models where agents face incomplete insurance markets for the idiosyncratic risk and credit constraints. These models with nominal frictions (HANK models) are now widely used to identify new transmission channels after various shocks or change in policy. The implications of these models for optimal stabilization policy are however unclear. Do we learn new insights about optimal fiscal and monetary stabilization policy compared to simpler complete-market (CM) models featuring a representative agent? Some recent papers conclude that the allocation, and sometimes the optimal dynamics of inflation, are close in HA

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and CM models (LeGrand et al. (2022), Challe (2020)). We show that this not the case and that HA models generate new mechanisms to think about monetary and fiscal stabilization policy. In a nutshell, these mechanisms rely on the time-varying demand for liquidity, which can be captured by simple statistics, but which crucially depend on the nature of the shock hitting the economy.

We show this property analyzing an heterogeneous agents models with both sticky prices and sticky wages and without capital. In this environment we solve jointly for optimal fiscal and monetary policy after a three types of shock: A TFP shock, a discount factor shock and a public spending shock. We consider a rich set of fiscal tools, as explained below. In each case, we solve for the optimal policy (using the same set of instruments) in the corresponding complete-market economy to identify the difference in allocation and instruments between the HA and CM economy.

In a first step, to provide a theoretical explanation of the difference between HA and CM models we focus on a simple environment which allows for analytical characterization of the difference between the allocation of HA and CM models. We consider a model with deterministic income fluctuations and credit constraints, as in Woodford (1990) and LeGrand and Ragot (2025). We solve for optimal policy in a flexible price environment where the tools are capital and labor tax and public debt, after three types of shock: a TFP shock, discount factor shock and a public spending shock. The dynamics of consumption and labor are exactly the same between CM and HA models after a TFP shock, but the dynamics is different after a discount factor shock. We difference in result comes from the fact that the normalized Lagrange multiplier on the credit constraints of agents is not time-varying after TFP shock, but it is time-varying after discount-factor shock. We called this normalized value the Marginal Value of Credit Constraint (MVCC) as it is the price a credit constrained agent would pay to relax the credit constraint by one unit. We show then show that a time-varying MVCC is a necessary condition for the allocation to be different in HA and CM economies. In other words, it isn't the presence of occasionally binding credit constraint which is driving a difference between CM and HA economies, but the fact that the intensity of the credit constraint is time varying. This property happens to be shock specific. It is the case for discount factor shocks, but much less so for TFP shock. We show that the MVCC is related to to the discount factor wedge, which is used by Nakajima (2005), Werning (2015), Acharya and Dogra (2021) and Berger et al. (2023) to compare aggregate allocation between HA and CM economies. An interest of the MVCC is that it has a microeconomic interpretation, as shown below

In a second step, we solve for optimal monetary and fiscal policy in the general incomplete-market model, with both sticky prices and sticky wages, and with a standard separable utility function. We consider a generalized Social Welfare Function to ensure that the steady state generates a relevant allocation (Auclert (2024)LeGrand and Ragot (2025)). First, we present a called complete fiscal system, composed of three labor taxes, a capital tax and public debt.

We theoretically show that when these tools are optimally time-varying optimal price and wage inflation is 0 after the three shocks (TFP, discount factor and public spending). This results extend equivalence result of Correia et al. 2008 or LeGrand et al., 2022 to an environment of both sticky prices and wages. From this theoretical benchmark, we then remove some fiscal instruments, considering a limited set of time-varying instruments, and we solve for optimal policy in the CM and HA economy for each aggregate shock. This strategy allows identifying the missing tools generating the highest welfare loss or a significant deviation from price and wage stability.

This strategy generates three sets of results.

[...]

Related literature. This paper belongs to the literature on optimal policy in heterogeneous agent model on one side, and on wage-price spirals on the other side. Deriving optimal policy in heterogeneous-agent models with aggregate shocks is a difficult theoretical and computational task. Some papers consider numerical methods to solve for optimal path of the instruments (Dyrda and Pedroni, 2022). Other papers rely on continuous-time techniques for the theoretical derivation of the first-order conditions of the planner (Nuño and Thomas, 2022 among others). Acharya et al. (2022) solve for optimal monetary policy using the tractability of the CARA-normal environment without capital. Bhandari et al. (2021) quantitatively solve for optimal policies in a new-Keynesian model with aggregate shocks. Yang (2022) solves for the optimal monetary policy by optimizing on the coefficients of a Taylor rule. McKay and Wolf (2022) derive a general quadratic-linear formulation to solve for optimal policy rules. McKay and Reis (2021) study optimal automatic stabilizers in the context of the optimal replacement rate. Their main focus is on the trade-off between insurance and incentives in the presence of an aggregate demand effect. The mechanism we identify is different in that it directly affects the gap between the real wage and the marginal productivity of labor.In this paper, we use the tools of LeGrand and Ragot (2022a) and the improvements of LeGrand and Ragot (2022b) to solve for optimal fiscal and monetary policy with aggregate shocks. The gain of this approach is to allows to easily solve for optimal policy with many tools and with various nominal frictions. On the theoretical side, the Lagrangian approach pioneered in Marcet and Marimon (2019) enables us to derive the first-order conditions of the Ramsey planner in an environment with both wage and price rigidities.

Regarding the literature on wage-price spirals, models including both price and wage stickiness have been studied in RA economies (Blanchard, 1986, Galí, 2015, chapter 6, or Blanchard and Gali, 2007 among others). Erceg et al. (2000) study optimal monetary policy in this environment. Chugh (2006) study both optimal monetary policy and an optimal labor tax. Recently, Lorenzoni and Werning (2023) analyze more deeply optimal policy and the real wage dynamics in this environment.

2 Understanding the difference between RA and HA economies in a simple environment

We provide in this section a simple flexible-price environment, where the difference between the optimal policy and allocations between HA and RA economies can be analytically characterized. The main simplifying assumption is to consider deterministic productivity fluctuations, as in Woodford (1990) and in LeGrand and Ragot (2025) among others. We here consider the flexible price economy and we first write the model in real terms to simplify the exposition. We discuss the underlying wage and price inflation dynamics is Section 2.4 below.

Production. The production function is simply $Y_t = Z_t L_t$. As prices are flexible, the real wage is $w_t = Z_t$.

The agents. The economy is populated two types of agents, denoted by A and B. A unit mass of agent A has a productivity 1 every odd period and a productivity 0 every even periods. A unit mass of agent B has a productivity 1 every even period and a productivity 0 every off periods. At each period there is thus a unit mass of agents with productivity 1, and income fluctuations are deterministic. The agents having a positive productivity are called "employed", and are identified with subscript e and agents with a 0 productivity are called called "unemployed", identified with subscript u.

The utility of agents is of the GHH type $U(c,l) = \log\left(c - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$, where c and l are individual consumption and labor supply respectively. Preferences can be time-varying, and we consider that all agents discount period $t+1$ utility at period t with a discount factor β_t,which can be time varying. More precisely we consider that there a value β, such that $\beta_t = \beta e^{u_t}$, where u_t follows a $AR(1)$ process after a period 0 shock.

$$u_0 \text{ given and } u_t = \rho^u u_{t-1} \text{ for } t \geq 1, \rho^u < 1.$$

An increase in β is thus a standard demand shock (Galí (2015)). At period 0, the discount factor is

$$\Theta_t = \Pi_{k=0}^{t} \beta_t, \tag{1}$$

which is β^t if β is constant.

The only friction in this economy is a credit constraint. It is assume that agents cannot borrow. We denote as $c_{e,t}, a_{e,t}, c_{u,t}, a_{u,t} \geq 0$ the consumption and saving of employed and unemployed agents respectively at period t. We denote as R_t and w_t the gross real post-tax interest rate and wage rate. the two budget constraints of e and u agents are

$$c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{2}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}, \tag{3}$$

The two Euler equations are

$$\left(c_{e,t}-\frac{l_{e,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)^{-1}\geq\beta R_{t+1}\left(c_{u,t+1}\right)^{-1}, \tag{4}$$

$$\left(c_{u,t}\right)^{-1}\geq\beta R_{t+1}\left(c_{e,t+1}-\frac{l_{e,t+1}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)^{-1}, \text{ with equality if } a_{u,t}>0, \tag{5}$$

Due to the GHH utility function the labor supply of employed agents is pinned down by the real wage

$$l_{e,t}=w_t^{\varphi} \tag{6}$$

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Government. The government issue a quantity of debt B_t. The financial marker equilibrium is simply

$$a_{e,t}+a_{u,t}=B_t. \tag{7}$$

The government raises capital τ_t^K and labor tax τ_t^L to finance interest payment on public debt. Denote as $\tilde{r}_t$ the pre-tax net rate interest rate. The gross post-tax real interest rate is $R_t=1+(1-\tau_t^K)\tilde{r}_t$. The post-tax wage rate is $w_t=\left(1-\tau_t^L\right)\tilde{w}_t$. The government budget constraint is $(1+\tilde{r}_t)\,B_{t-1}\leq B_t+\tau_t^K\tilde{r}_t\left(a_{e,t}+a_{u,t}\right)+\tau_t^L\hat{w}_tL_t$. It can be written in post-tax term as

$$R_tB_{t-1}=\left(Z_t-w_t\right)L_t+B_t \tag{8}$$

2.1 Some benchmark economies : representative-agent, complete-market and no-policy economies

We first present the allocation in three simple economies, as a benchmark.

Representative agent. The representative-agent economy is particularly simple to represent. The first-best can be achieved by setting labor and capital tax to 0. With the GHH assumption, the labor supply is $L_t^{RA}=Z_t^{\varphi}$. An optimal consumption is $C_t^{RA}=Z_t^{1+\varphi}$. Tt is

$$C_t^{RA}=Z_t^{1+\varphi}$$

At the steady-state $Z=1$, $C^{RA}=1$. Although the state is not raising resources, one can compute the Lagrange multiplier on the budget of the state, which is simply the marginal utility of the representation agent, as there are no distortions :

$$\mu_t^{RA}=\frac{1}{C_t^{RA}-\frac{\left(L_t^{RA}\right)^{1+\frac{1}{\varphi}}}{1+1/\varphi}}=\frac{1+\varphi}{Z_t^{1+\varphi}}$$

Complete market economy. In the complete market economy, the ratio of marginal utilities is constant between employed and unemployed agents and depends on exogenous initial conditions. In this case, one can easily show that aggregate consumption, denoted as C_t^{CM}, is $C_t^{CM} = C_t^{RA} = Z_t^{1+\varphi}$.

No Policy economy. If the government doesn't intervene in this economy, then $B_t = 0$, $w_t = Z_t$ such that labor taxes are $\tau_t^L = 0$. The allocation is also simple to derive. The employed agent is working such that $L_t = Z_t^{\varphi}$ and it consumes its income, as there is no store of value. Hence, $c_{e,t} = Z_t^{1+\varphi}$and $c_{u,t} = 0$. There is no consumption smoothing, the aggregate consumption in this economy, denoted as C_t^{NP} is

$$C_t^{NP} = C_t^{RA}.$$

As a consequence, these three economies generate the same aggregate consumption path after both supply shocks (Z_t) and demand shocks β_t, as demand shock is not affecting consumption in any of the three economies.

We now study how optimal fiscal policy affects risk sharing and aggregate consumption.

2.2 The incomplete-market economy with optimal policy

We first consider an economy with flexible prices, and discuss inflation dynamics below. For a given path for $(\beta_t, Z_t)_{t=0\ldots\infty}$, using the cumulative discount factor $\Theta_t = \Pi_{k=0}^t \beta_t$, the program of the Utilitarian planner is

$$\max_{(c_{e,t}, c_{u,t}, a_{e,t}, a_{u,t}, l_{e,t} B_t, A_t, R_t, w_t)} \sum_{t=0}^{\infty} \Theta_t \left(\log \left(c_{e,t} - \frac{l_{e,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) + \log \left(c_{u,t} \right) \right) \tag{9}$$

$$c_{e,t}, c_{u,t}, a_{e,t}, a_{u,t}, l_{e,t}, l_{u,t} \geq 0. \tag{10}$$

The planner maximizes the aggregate welfare criterion (9) considering the relevant discount factor Θ_t embedding possibly time varying β_t, subject to the following: the constraints (2)–(8), which guarantee the optimality of individual choices (budget constraints, Euler equations and labor FOC with GHH utility function, respectively); the resource constraint (8) where we use the fact that $L_t = l_{e,t}$; the financial market clearing condition (7); the credit constraints (10); and the positivity constraints ($c_{e,t}, c_{u,t}, a_{e,t}, a_{u,t}, l_{e,t}, l_{u,t} \geq 0.$).

This allocation is actually easy to derive. Indeed, we prove that the planner doesn't want to increase public debt to the level for which credit constrained would not bind for u-agents. Indeed, this would imply to high a distorting labor tax. As a consequence, the credit constraint is binding for unemployed agents. Then using the labor supply (6), the program can be considerably simplified. We derive all proofs in Appendix ????, and provide only main results in this Section.

Denote aggregate consumption in this economy as $C_t^{HA} = c_{u,t} + a_{u,t}$, where HA stands for heterogeneous-agent with optimal fiscal policy. In we denote as $\tilde{x}_t$ as the proportional deviation of variable x_t to its steady state value. The next proposition compared HA and RA economies.

Proposition 1 *For any value of Z_t and β_t the optimal allocations in the HA and RA case satisfy*

$$\frac{C_t^{HA}}{C_t^{RA}} = \left(\frac{1+\varphi\left(1+\beta_t\right)}{\left(\frac{1}{2}+\varphi\right)\left(1+\beta_t\right)}\right)^{-\varphi}$$

This proposition states the main result of this Section.

Supply shocks Z_t only. First, if the economies are hit only by supply shocks, then $\beta_t = \beta$. In this case, $\frac{C_t^{HA}}{C_t^{RA}}$ is constant from Proposition 1. This implies that $\tilde{C}_t^{HA} = \tilde{C}_t^{RA}$. In wards, although RA and HA economy have different steady states, their aggregate behavior is similar as, proportional deviations after supply shocks are similar.

Demand shock β_t only. Then the economy is hit by demand shock, the ratio $\frac{C_t^{HA}}{C_t^{RA}}$ is time-varying and RA and HA economies behave differently $\tilde{C}_t^{HA} \neq \tilde{C}_t^{RA}$.

As a consequence, only demand shocks generate a difference between the allocations of HA and RA economies. Obviously, for both supply and demand shocks, the paths of the instruments are different between RA and HA economies. Indeed, in the RA economy $B_t = \tau_t^L = 0$ to implement the first-best equilibrium at each period, and the path of the instruments is moving for both demand and supply shocks. As a consequence, the difference in the allocations between HA and RA is here the object of interest it captures the difference between HA and RA for the two types of shocks.

2.3 Three statistics

To understand the effect of supply and demand shocks when optimal policy is implemented we introduce three some useful statistics.

The Marginal value of the Credit Constraints (MVCC)

First, define as ν_t the Lagrange multiplier of the credit constraint of u-agent. It is $\nu_t = U_c(c_{u,t}, 0) - \beta R_{t+1} U_c(c_{e,t+1}, l_{e,t+1})$. In words, it is the difference between the current and discounted expected marginal utilities of agents u.

Define the Marginal value of Credit Constraints (MVCC) as

$$MVCC_t := \frac{\nu_t}{U_c(c_{u,t}, 0)}$$

It is the the Lagrange multipliers ν_t normalized by the current marginal utility of agents u. The MVCC has an simple economic interpretation: It is the maximum fee that agents u would accept to pay to be able to borrow one additional unit. It is 0 if credit constraints don't bind. Using the definition of ν_t, one can write

$$U_c(c_{u,t},0)=\beta_t\frac{R_{t+1}}{1-MVCC_t}U_c(c_{e,t+1},l_{e,t+1})$$

The $MVCC$ is renormalization of the discount factor of credit constrained agents. When the $MVCC$ is constant then the real interest rate R_{t+1} is a sufficient statistics for the growth rate of marginal utility and consumption, and then of consumption for unemployed agents (as for employed agents who have an Euler equation). The MVCC is constant, the consumption dynamics is the same and depends only on the real interest rate. As a consequence, it is the dynamic properties of the MVCC which matter for the difference between RA and HA, not the steady state value.

The MVCC can be explicitly computed in this economy it is

$$MVCC_t=1-\frac{\left(1+\varphi\left(1+\beta_{t+1}\right)\right)\left(1+\varphi\left(1+\beta_t\right)\right)}{\left(1+2\varphi\right)^2}$$

One can check that the $MVCC_t$ is constant when β is constant and thus doesn't vary with technology shock Z_t. This explains why RA and HA economies behave the same way after supply shocks.

The discount factor wedge (DFW)

Following Nakajima (2005), Werning (2015), Acharya and Dogra (2021) and Berger et al. (2023), it is known that the allocation of HA-models can be compared to the one of RA-models where some wedges are introduced to reproduce the path of relevant variables such as consumption and output. The *discount factor wedge* (DFW) is the wedge such that the allocation and interest rate in the HA economies would be an equilibrium in the RA economies if the discount factor is multiplies by a wedge β_t^{wedge}. It thus allows to reproduce allocation and prices obtained in the HA economy as an equilibrium path in the RA economy, and to assess distortions in the saving incentives due to market incompleteness. More formally, this wedge solves the Euler equation of the representative agent:

$$\left(C_t^{HA}-\frac{l_{HA,t}^{1+1/\varphi}}{1+1/\varphi}\right)^{-1}=\beta_t^{wedge}\beta_t R_{t+1}^{HA}\left(C_{t+1}^{HA}-\frac{l_{HA,t+1}^{1+1/\varphi}}{1+1/\varphi}\right)^{-1}$$

After some algebra, one finds that the DFW, which is $\beta_t^{wedge}-1$

$$DFW_t=\beta_t^{wedge}-1=\frac{\left(1+1/\varphi\right)\left(1+\varphi\left(1+\beta_{t+1}\right)\right)-\left(1/2+\varphi\right)\left(1+\beta_{t+1}\right)}{\left(1+1/\varphi\right)\left(1+\varphi\left(1+\beta_t\right)\right)-\left(1/2+\varphi\right)\left(1+\beta_t\right)}\times\frac{1+2\varphi}{1+\varphi\left(1+\beta_{t+1}\right)}-1$$

The wedge DFW_t concerns the aggregate consumption of all agents (constrained and unconstrained). However, as the unconstrained agents are on their Euler equation, movements of the DFW is mainly driven by unconstrained agents. In the quantitative model below, we report both the MVCC and the DFW and we check that they behave the same way[1]. At the steady state, the relationship between the MVCC and the DFW is much simpler. We find $MVCC = 1 - (1 + DFW)^{-2}$, such that the credit constraint doesn't bind when the two wedge is 0 (ie. HA and CM coincides) : $MVCC = 0$ when $DFW = 0$.

The Marginal value of Public Fund (MVPF)

Another statistics is the Lagrange multiplier μ_t on the budget of the state. This multiplier is often the called the marginal value of public fund (MVPF) in public finance (Ferey et al. (2024) among others). If the planner as non-distorting tools (such as lump-sum transfer to each agent), one can show that MVPF is equal to the marginal productivity of each agent. When the MVPF is higher is means that raising resources distort the economy. As consequence, the MVPF captures the distortions generated by the tools of the planner, and thus depends on the set of fiscal tools available to the planner.

One can show that the MVPF of the RA and HA economies are

$$\frac{\mu_t^{HA}}{\mu_t^{RA}} = (1 + 2\varphi)^{-\varphi} \left(\frac{1 + \beta_t}{2}\right)^{-1-\varphi}$$

The MVCC focus on the frictions at the micro level, generating difference in consumption dynamics, whereas the MPVF captures the overall distortions induced by the specific instruments. As the discount factor wedge the $MPVF$ capture global differences.

Other statistics

We also report other standard statistics (such as liquidity premium or consumption inequalities), but they appear to be hard to interpret to understand the different in optimal policies between HA and RA economies.

Liquidity premium. First, since Aiyagari (1994) a standard statistics in HA model is the liquidity premium (LP), which is the difference between the real interest rate in the complete market and the HA models, which is positive at the steady state. With our timing it is $LP_t := \frac{1}{\beta_{t-1}} - \frac{1}{R_t}$. It has the following complex expression

$$LP_t = \frac{1}{\beta_{t-1}} \left(1 - \frac{1}{1 + 2\varphi} \frac{(1 + \varphi (1 + \beta_t))^{-\varphi}}{(1 + \varphi (1 + \beta_{t-1}))^{-(1+\varphi)}} \left(\frac{1 + \beta_{t-1}}{1 + \beta_t}\right)^{-\varphi} \left(\frac{Z_t}{Z_{t-1}}\right)^{1+\varphi}\right)$$

[1]The relationship between the intensity of the credit constraint and the discount factor wedge is discussed in Guerrieri and Lorenzoni (2017).

The LP moves in a non trivial way with both demand and supply shocks, and it is thus hard to interpret. At the steady state however, one finds a simple relationship between MVCC and LP:

$$MVCC = 1 - (1 - \beta \times LP)^2 = 1 - (1 + DFW)^{-2}$$

Hence, the MVCC is also null when the LP is null.

Consumption inequality. Consumption inequality is also not discriminating between types of shocks. Indeed, we have $\frac{c_{e,t}}{c_{u,t}} = 1 + 2\varphi$ for both supply and demand shocks in the HA economy.

Statistics and optimal policies

The previous statistics are useful to understand the difference between the HA and the RA allocations when the optimal policy is implemented, which is the focus of the paper. However, they are not sufficient to determine optimal policies in the HA environment. To see that, consider the value of these statistics in the No Policy allocation. One easily finds $MVCF_t^{NP} = +\infty$ as low productivity agent don't consume and their marginal utilty is infinite. In addition, s $\beta_t^{wedge,NP} = 1$ as the aggregate consumption and labor is the same as in the RA economy. The nature and the welfare gain of optimal policies require solving the full model.

2.4 Intuition for the role of inflation using the simple model

Finally, the intuition for the role of price and wage inflation can be provided by two key-equations in this simple model, considering underlying inflation dynamics in this flexible price environment. In the monetary economy, the nominal interest rate set by monetary authorities is denoted i_t. The price level is P_t and $\pi_t^P = \frac{P_t - P_{t-1}}{P_{t-1}}$ is the net inflation rate. The nominal wage is W_t, and $\pi_t^W = \frac{W_t - W_{t-1}}{W_{t-1}}$ is the wage inflation. The real post-tax interest rate is then

$$r_t = \left(1 - \tau_t^K\right) \frac{i_{t-1} - \pi_t^P}{1 + \pi_t^P}$$

The dynamics of the post tax real wage $w_t = \left(1 - \tau_t^L\right) W_t / P_t$ is

$$\frac{1 + \pi_t^W}{1 + \pi_t^P} = \frac{w_t / \left(1 - \tau_t^L\right)}{w_{t-1} / \left(1 - \tau_{t-1}^L\right)}$$

These two equations capture the two main effects of inflation.

Capital tax. When capital tax is not available, a change inflation allows to affect the real interest rate. it is thus a costly substitute for missing capital tax.

Labor tax. The relative inflation dynamics $\frac{1+\pi_t^W}{1+\pi_t^P}$ will depend on both the real wage rate and on the use of fiscal tools τ_t^L due to improve consumption smoothing. Differences in fiscal policy between HA and RA agents will translate into inflation dynamics.

When prices are flexible (the environment of the current Section) π_t^P and π_t^W are a residual. Under the sticky price and wages, the cost to move π_t^W and π_t^P will affect the optimal evolution of fiscal policy to contribute to reduce price and wage volatility, by these two channels, which have already been identified in in Auclert (2019). [2]

Unfortunately introducing explicitly sticky prices and wages in this simple model doesn't useful analytical results. We now consider the general model to deliver additional theoretical and numerical results.

3 The General model

In the general model, we relax many of the simplifying assumptions of the previous Section, and we introduce both sticky prices and sticky wages, together with a rich fiscal structure.

We consider a discrete-time economy populated by a continuum of size one of ex-ante identical agents. These agents are assumed to be distributed along a set J, with the non-atomic measure ℓ: $\ell(J) = 1$.[3]

3.1 Risk

We assume that the agents face an idiosyncratic productivity risk. The productivity process, denoted y, is assumed to take value in a finite set $\mathcal{Y}$ and to follow a first-order Markov chain with transition matrix $\boldsymbol{\pi} = (\pi_{yy'})_{y,y'}$. With wage w and labor supply l, an agent with productivity y earns the labor income wyl. In each period, the fraction of agents with productivity y is constant and denoted by n_y. We normalize average productivity to 1, i.e., such that $\sum_y n_y y = 1$. The history of idiosyncratic productivity shocks up to date t for an agent i is denoted by $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\} \in \mathcal{Y}^{t+1}$, where $y_{i,\tau}$ is the date-τ productivity. The measure of idiosyncratic histories up-to-date t, denoted by θ_t, can be computed using the initial distribution and the transition matrix.

As before, the discount factor is $\beta_t = \beta e^{u_t}$, where u_t follows a $AR(1)$ process after a period 0 shock.

$$u_0 \text{ given and } u_t = \rho^u u_{t-1} \text{ for } t \geq 1, \rho^u < 1.$$

and $\Theta_t = \Pi_{k=0}^t \beta_t$.

Aggregate risks. In addition to the previous idiosyncratic risk, agents face an aggregate supply shock, affecting either the economic TFP, denoted by Z_t, or discount factor shocks β_t, or

[2]The channels presented here are the earnings heterogeneity channel induced by the effect of price and wage inflation on the real wage rate, and the real interest rate exposure channel induced a Fisher channel. The interest of this Section is to identify the interactions with tax rates.

[3]We follow Green (1994) and assume that the law of large numbers holds.

public spending shocks[4] G_t.

These three aggregate shocks $(\beta_t, Z_t, G_t)_{t\geq 0}$ are persistent but are known at period 0, and should thus be considered as MIT shocks.

3.2 Preferences

Households are expected-utility maximizers endowed with time-separable preferences and a constant discount factor $\beta \in (0,1)$. In each period, households enjoy utility $U(c,l)$ from the consumption c of the unique consumption good of the economy and suffer from the disutility of providing the labor supply l. We further assume that in each period, the instantaneous utility is separable in consumption and labor: $U(c,l) = u(c) - v(l)$, where $u, v : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously differentiable and increasing. Furthermore, u is concave, with $u'(0) = \infty$, and v is convex.

3.3 Labor taxes

For the sake of generality, and for theoretical reasons which we develop in Section 3.8 below, we introduce a rich set of four linear taxes. We here present the two labor taxes, and introduce income and capital tax below.

First, we assume that unions bargain over the nominal wage rate, denoted by $\hat{W}_t$. Workers pay a linear labor tax τ_t^L on this income such that their post-tax nominal wage is $(1-\tau_t^L)\hat{W}_t$. Second, firms pay an additional labor tax, τ_t^S, which implies a wedge between the labor cost per efficient unit of labor, $\tilde{W}_t$, paid by firms and the wage $\hat{W}_t$ bargained by workers. This additional tax can be thought of as an employer social contribution that does not appear on the payroll of workers. Formally, the labor cost $\tilde{W}_t$, the bargained wage $\hat{W}_t$ and the tax τ^S verify the following relationship: $\hat{W}_t = (1-\tau_t^S)\tilde{W}_t$. The tax τ_t^S will have an effect on labor demand that will be internalized by unions in their bargaining strategy. Similarly, the tax τ_t^L will have an effect on labor income that will also be internalized. The difference between the two taxes is that τ_t^S has a direct effect on employment for a given bargained wage $\hat{W}_t$ but not on the wage W_t, whereas τ_t^L has a direct effect on the wage W_t for a given bargained wage $\hat{W}_t$, but no direct effect on employment.[5]

3.4 Production

The specification of the production sector follows the New-Keynesian literature on price stickiness, adapted to the previous tax structure. The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y_{j,t}^f)_j$ from different sectors

[4]We show in Section ?? that a shock on energy price is equivalent to a negative shock to TFP.

[5]We call direct effect the partial equilibrium effect of each variable. In general equilibrium (with endogenous income), these taxes obviously affect all variables through price variations.

indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with an elasticity of substitution, denoted ε_P:

$$Y_t = \left[\int_0^1 y_{j,t}^{f\ \frac{\varepsilon_P - 1}{\varepsilon_P}} dj\right]^{\frac{\varepsilon_P}{\varepsilon_P - 1}}.$$

For any intermediate good $j \in [0,1]$, the production $y_{j,t}^f$ is realized by a monopolistic firm and sold at price $p_{j,t}$. Z_t is aggregate labor productivity. It is affected at period 0 by a shock ϵ_0^Z and it follows a AR(1) process. $Z_t = e^{z_t}$,with

$$z_0 = 1 + \epsilon_0^Z \text{ and } z_t = \rho^Z z_{t-1} \text{ for } t \geq 1, \rho^Z < 1.$$

Intermediate firms face a quadratic price adjustment cost à la Rotemberg when setting their price. Following the literature, the price adjustment cost is proportional to the magnitude of the firm's relative price change and equal to $\frac{\psi_p}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2$. Denoting the price inflation rate as $\pi_t^P = \frac{P_t}{P_{t-1}} - 1$ where P_t is the implied price index, we obtain the standard equation characterizing the Phillips curve after standard derivation relegated in Appendix

$$\pi_t^P(1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}(m_t - 1) + \beta_t \mathbb{E}_t\left[\pi_{t+1}^P(1 + \pi_{t+1}^P)\frac{Y_{t+1}}{Y_t}\right], \tag{11}$$

where $Y_t = Z_t L_t$.

3.5 Labor market: Labor supply and Union wage decision

Following the New Keynesian sticky-wage literature, labor hours are supplied monopolistically by unions (Erceg et al. (2000)Chugh (2006) Hagedorn et al. (2019) or Auclert et al., 2022 among others). There is a continuum of unions of size 1 indexed by k and each union k supplies L_{kt} hours of labor at date t with nominal wage $\hat{W}_{kt}$. Each union k sets its wage $\hat{W}_{kt}$ so as to maximize the intertemporal welfare of its members subject internalizing the demand by firms. We assume the presence of quadratic utility costs related to the adjustment of the nominal wage and equal to $\frac{\psi_W}{2}(\hat{W}_{kt}/\hat{W}_{kt-1} - 1)^2 dk$. The objective of union k is thus:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \Theta_s \int_i \left(U(c_{i,s}, l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2\right)\ell(di),$$

This maximization will provide the New-Keynesian wage-Phillips curve:

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1 - \tau_t^L)\hat{w}_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}} L_t + \beta \mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W + 1)\right], \tag{12}$$

where $\pi_t^W = \frac{\hat{W}_t - \hat{W}_{t-1}}{\hat{W}_{t-1}}$ is the wage inflation rate of the wage bargained by unions.n and $\hat{w}_t = \hat{W}_t / P_t$ is the real pre-tax wage, as $\hat{W}_t$ is the bargained nominal wage.

3.6 Assets

The only asset is nominal public debt, whose supply size is denoted by B_t at date t, and which pays off the pre-determined before-tax nominal interest rate i_{t-1}. Public debt is issued by the government and assumed to be default free. The financial market clearing implies that the net total savings of households, denoted A_t, must equal public debt:

$$A_t = B_t. \tag{13}$$

The corresponding real before-tax (net) interest rate for public debt, denoted by $\tilde{r}_t$, is defined by:

$$\tilde{r}_t = \frac{1 + i_{t-1}}{1 + \pi_t^P} - 1. \tag{14}$$

3.7 Agents' program

Each agent enters the economy with an initial endowment of public debt $a_{i,-1}$ and an initial productivity level $y_{i,0}$. The joint initial distribution over public debt and productivity levels is denoted Λ_0. In later periods, each agent learns her productivity level $y_{i,t}$, supplies labor, and earns her savings payoffs. Since the labor supply L_t is chosen by unions, the labor income is $(1 - \tau_t^L)\hat{w}_t y_{i,t} L_t$. The before-tax real financial payoff amounts to $\tilde{r}_t a_{i,t-1}$.

We assume that agents pay two other taxes. First, a capital tax $\hat{\tau}_t^K$ is levied on interest payment and implies a net asset payoff $(1 - \tau_t^K)\tilde{r}_t a_{i,t-1}$. Second, an income tax τ_t^E is levied on total labor income, $\tau_t^E(1 - \tau_t^L)\hat{w}_t y_{i,t} L_t$. The latter income tax τ_t^E is not internalized by the unions as each union as a marginal contribution to total income. As a consequence, the post-tax total income is equal to $(1 - \tau_t^E)(1 - \tau_t^L)\hat{w}_t y_{i,t} L_t$.

Agents earn this net total income and use it together with their past savings to consume $c_{i,t}$ and save $a_{i,t}$. Their budget constraint can be expressed as follows:

$$c_{i,t} + a_{i,t} = a_{i,t-1} + (1 - \tau_t^K)\tilde{r}_t a_{i,t-1} + (1 - \tau_t^E)\big((1 - \tau_t^L)\hat{w}_t y_{i,t} L_t\big). \tag{15}$$

To simplify the previous notation, we define the post-tax real interest and wage rates as:

$$r_t = (1 - \tau_t^K)\tilde{r}_t, \tag{16}$$

$$w_t = (1 - \tau_t^E)(1 - \tau_t^L)\hat{w}_t = (1 - \tau_t^E)(1 - \tau_t^L)(1 - \tau_t^S)\tilde{w}_t. \tag{17}$$

As the $\hat{W}_t / P_t = w_t / (1 - \tau_t^E)(1 - \tau_t^L)$, we have the law of motion of the post-tax real wage as a function of inflation and taxes

$$(1+\pi_t^W)\frac{w_{t-1}}{(1-\tau_{t-1}^L)\,(1-\tau_{t-1}^E)}=\frac{w_t}{(1-\tau_t^L)\,(1-\tau_t^E)}(1+\pi_t^P) \tag{18}$$

The agent's program can be finally be written as:

$$\max_{\{c_{i,t},a_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_t \left(U(c_{i,t}, L_t)\right), \tag{19}$$

$$c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t, a_{i,t}, \tag{20}$$

and subject to the credit constraint $a_{i,t} \geq -\underline{a}$, and the consumption positivity constraint $c_{i,t} > 0$. The notation $\mathbb{E}_0$ is an expectation operator over both idiosyncratic and aggregate risks. The solution of the agent's program is a sequence of functions, defined over $([-\bar{a}; +\infty) \times \mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$ and denoted by $(c_t, a_t)_{t\geq 0}$, such that:[6]

$$c_{i,t} = c_t((a_{i,-1}, y_{i,0}), y_i^t, z^t),\ a_{i,t} = a_t((a_{i,-1}, y_{i,0}), y_i^t, z^t). \tag{21}$$

For the sake of simplicity, we will keep using the notation with the i-index. Denoting by $\nu_{i,t}$ the discounted Lagrange multipliers of the credit constraint, the Euler equation corresponding to the agent's program (19) is:

$$u'(c_{i,t}) = \beta_t \mathbb{E}_t\Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}. \tag{22}$$

with the complementary slackness condition

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0 \tag{23}$$

The MVCC

As in Section 2, one can easily introduce the MVCC. For agents i, it is $MVCC_{i,t} = \nu_{i,t}/u'(c_{i,t})$. Such that, for all agents:

$$u'(c_{i,t}) = \beta_t \mathbb{E}_t\left[\frac{1 + r_{t+1}}{1 - MVCC_{i,t}} u'(c_{i,t+1})\right].$$

In the HA economy, there is a distribution of $MVCC$. When agents are not constrained $MVCC_{i,t} = 0$, and $0 < MVCC_{i,t} \leq 1$ otherwise. In quantitative Section, we will comment on the average $MVCC$ over agents.

[6]See e.g. Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2018) for a proof of the existence of such functions.

3.8 Government and market clearing conditions

The government has to finance an exogenous public good expenditure G_t, by raising a quite large number of taxes and by issuing one-period riskless public debt. G_t is affected at period 0 by a shock ϵ_0^G and it follows a AR(1) process. $G_t = e^{g_t}$,with

$$g_0 = 1 + \epsilon_0^G \text{ and } g_t = \rho^G g_{t-1} \text{ for } t \geq 1, \rho^G < 1.$$

The government raises four linear taxes: (i) a tax τ_t^S based on labor cost $\tilde{w}_t$ and paid by employers, (ii) a tax τ_t^L based on bargained wage $\hat{w}_t$ and paid by workers, and finally (iii) a tax τ_t^E based on total income and paid by workers (iv) a capital tax τ_t^K. Importantly, the three labor instruments (τ_t^L, τ_t^S and τ_t^E) are independent and not redundant, on the one hand, τ_t^S creates a wedge between the labor cost and the bargained wage, while τ_t^L and τ_t^E create wedges between the bargained wedge and the net wage. On the other hand, τ_t^L is internalized by unions, while τ_t^E is not. These three taxes will play on different margins and will allow us derive our equivalence result below. Hence, they should be understood as theoretical tools needed to generate price and wage stability. Each tax will be removed in turn to consider more realistic fiscal settings and to assess how each fiscal instrument contributes to inflation volatility. In addition to capital and labor taxes and to public debt, the government also fully taxes the firms' profits, Ω_t, which limits the distortions implied by profit distribution.

We can now express the government budget constraint. The government has to finance public spending and the repayment of past public debt. Its resources consist of all labor taxes, capital taxes, corporate profits, and newly issued public debt. We obtain:

$$\begin{aligned} G_t + \frac{1 + i_{t-1}}{1 + \pi_t^P} B_{t-1} \leq \Omega_t + B_t + \tau_t^E (1 - \tau_t^L) \hat{w}_t L_t \\ + \tau_t^K \tilde{r}_t \int_i a_{i,t-1} \ell(di) + \tau_t^L \hat{w}_t L_t + \tau_t^S \tilde{w}_t L_t. \end{aligned}$$

We can simplify the previous government budget constraint using the financial market clearing (13), the post-tax interest rate $\tilde{r}_t$ (14) and the post-tax rate definitions (16) as:

$$G_t + r_t B_{t-1} + w_t L_t \leq \Big(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \Big) Y_t + B_t - B_{t-1}, \tag{24}$$

We finally express the financial market clearing condition and the economy resource constraints:

$$\int_i a_{i,t} \ell(di) = B_t, \tag{25}$$

$$\int_i c_{i,t} \ell(di) + G_t = \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t. \tag{26}$$

Equilibrium definition. We can finally formulate our definition of competitive equilibrium.

Definition 1 (Sequential equilibrium) *For any exogenous paths of TFP* $(Z_t)_t$ *and of public spending* $(G_t)_t$*, and discount factor* (β_t) *a sequential competitive equilibrium is a collection of individual allocations* $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$*, of aggregate quantities* $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$*, of price processes* $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$*, of monetary policy* $(i_t)_{t\geq 0}$*, fiscal policies* $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$*, and inflation dynamics* $(\pi_t^W, \pi_t^P)_{t\geq 0}$ *such that, for an initial wealth and productivity distribution* $(a_{i,-1}, y_{i,0})_{i\in\mathcal{I}}$*, and for an initial value of public debt verifying* $B_{-1} = \int_i a_{i,-1}\ell(di)$*, we have:*

1. *given prices, the allocations* $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program (19)–(20);*
2. *financial, and goods markets clear at all dates: for any* $t \geq 0$*, equations (25) and (26) hold;*
3. *the government budget is balanced at all dates: equation (24) holds for all* $t \geq 0$*;*
4. *firms' profits* Ω_t *and the mark-up* m_t *are consistent with firms profit maximization.*
5. *the price inflation path* $(\pi_t^P)_{t\geq 0}$ *is consistent with the price Phillips curve (11), while the wage inflation path* $(\pi_t^W)_{t\geq 0}$ *is consistent with the wage Phillips curve (33);*
6. *the real and nominal rates* $(\tilde{r}_t, i_t)_{t\geq 0}$ *verify (14);*
7. *post tax rates* $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$ *are defined in equations (16)–(17).*

Social Welfare Function. Following LeGrand et al. (2022), we assume that the planner maximizes a generalized social welfare function, where the weights on each period utility can depend on the current productivity of the agent. The objective of the planner is thus:

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \Theta_t \int_i \omega(y_t^i) \left(U(c_t^i, l_t^i) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right]. \tag{27}$$

This expression embeds the utilitarian case, where $\omega(y) = 1$ for all y. This generalization of the Standard Social Welfare Function is now used either in quantitative work, such as (LeGrand et al., 2022, McKay and Wolf, 2022), or in more theoretical investigations, as a deviation from the utilitarian case (Dávila and Schaab, 2022). It will be used to ease the simulations and comparisons of economies in Section 5.

We assume that the economy starts from the steady-state situation where the fiscal system is optimally determined. Then in period 0, the economy is hit either by a demand shock or a supply shock. The whole paths of these shocks is known at period 0, and the planner sets optimally its available instruments under commitment.

We have introduced five fiscal instruments $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$. This rich fiscal system is also a theoretical device to understand distortions in the HA economy with both price and wage stickiness. As will be clear below, this fiscal system is the minimal one such that there is no deviation from price stability in all cases. In what follows, we consider different fiscal systems, where only some fiscal instruments can be optimally time-varying to smooth the effect of the shock, but not necessarily all of them. More precisely, we solve for optimal monetary policy considering a set $S_t \subset (\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$ of fiscal instruments. For any instruments I_t not in S_t, we assume $I_t = I_{ss}$.

We can define the notion of Ramsey equilibrium using this notion of social welfare function.

Definition 2 (Ramsey equilibrium) *For a given path of $(\beta_t, G_t, Z_t)_{t=0....\infty}$, a Ramsey equilibrium with all instruments is the path of monetary policy $(i_t)_{t\geq 0}$, fiscal instruments $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$, which selects a sequential equilibrium defined in 1, which is maximizing the social welfare function (27).*

A steady-state Ramsey equilibrium is a Ramsey equilibrium where aggregate real variables $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, prices $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ are constant.

Definition 3 (Ramsey steady state) *A steady-state Ramsey equilibrium is a Ramsey equilibrium where aggregate real variables $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, prices $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ are constant. The value of the instruments are denoted as $(\tau_{ss}^L, \tau_{ss}^S, \tau_{ss}^E, \hat{\tau}_{ss}^K, B_{ss})$.*

Finally, we can define our optimal policy with a limited set of instruments. Note that we consider that public is always time-varying to avoid considering too many cases. Time-varying public debt seems theoretically and empirically in this environment.

Definition 4 (Ramsey equilibrium limited) *For a given path of $(\beta_t, G_t, Z_t)_{t=0....\infty}$, a Ramsey equilibrium with a limited number of instruments is the path of monetary policy $(i_t)_{t\geq 0}$, fiscal instruments $S \subset (\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K)_{t\geq 0}$, and the instruments $I_t = I_{ss}$ if $I_t \notin S$, which selects a competitive equilibrium, which is maximizing the social welfare function (27).*

To determine the optimal steady state value of the instruments, we first solve the Ramsey model without aggregate shock to get $(\tau_{ss}^L, \tau_{ss}^S, \tau_{ss}^E, \hat{\tau}_{ss}^K)$, and then we solve for the optimal dynamics of other instruments. As a consequence, note that all economies with have the same steady-state equilibrium whatever the set of fiscal tools S, which is used in the transition[7].

3.9 The complete-market economy

The complete market economy is very standard. A representative agent maximizes utility in an economy without financial constraints. The planner has distorting tools and public debt to finance the public spending. The provide the algebra in Appendix ???.

[7]In the numerical simulations below, we indeed find that the economy always goes back to the initial steady state after transitory aggregate shock.

4 Optimal policies with Heterogeneous Agents

4.1 Characterizing the Ramsey allocation

We now consider heterogeneous-agent economies and derive optimal policies for both supply and demand shocks, for any set of fiscal instruments S. The Ramsey planner's program is

$$\max_{\left(S,w_t,r_t,L_t,B_t,\pi_t^P,\pi_t^W,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_i \omega(y_t^i)\left(u(c_t^i) - v(L)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right], \quad (28)$$

Subject to the budget of the State (24), the individual budget constraints (20), the individual Euler equation (22) and the slackness condition (23), the price Phillips curve (11), the Wage Phillips curve (12), the real wage dynamics (18), positivity constraints $c_t^i, l_t^i \geq 0$, given initial conditions a_{-1}^i, and finally the constraint on the instruments $I_t = I_{ss}$ if $I_t \notin S$. With full program is given in Appendix ???; where we derive the first-order conditions of the planner. As in LeGrand and Ragot (2025), we use some aspect of Marcet and Marimon (2019) to factorize the Lagrangian. On a technical note, the factorization of the price and wage Phillips curve is easy, as they are both Euler equations of firms and unions.

This economy faces different frictions, which are worth summarizing. The monetary economy features two sets of market imperfections. The first set is related to the goods market. Intermediary firms enjoy a monopoly power, which implies a price markup m_t that can differ from one. There is also a Rotemberg cost for price adjustment, which prevents firms from freely setting their price. Note that the good market imperfections are complementary: one vanishes when the other is absent, as can be seen from the price Phillips curve (11). The second set of imperfections is related to the labor market. The union implies that the labor supply of agents is not set optimally, while the Rotemberg cost for wages prevents unions from freely setting wages. Note that in the absence of Rotemberg cost, the labor supply still remains sub-optimal, as it remains set at the union level. Without Rotemberg cost, the equation characterizing the choice of the labor supply (common to all agents) would be $v'(L_t) = w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)$, while it would be $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$, if agents were able to choose their individual labor supply $l_{i,t}$. This sub-optimal common labor choice will play a major role in our equivalence results below.

To get more analytical insight about the optimal allocation in the HA economy, it is possible to provide the expression the *social valuation of liquidity (SVL) for agent* i, which is the the value for the planner to transfer one extra unit of consumption good to agent i in period t. In LeGrand and Ragot (2025), we show that this statistics simplifies the derivation of the first order of the planner, and that it is the same statistics as the *Generalized Social Marginal Welfare Weights* (GSMWW) introduced by Saez and Stantcheva (2016).

More precisely, we denote by $\beta^t \lambda_{i,t}$ the Lagrange multipliers of the Euler equations (22 of agent i at date t. The Lagrange multiplier of the government budget constraint is $\beta^t \mu_t$. (24), and $\gamma_{W,t}$ is the Lagrange multiplier on the wage Phillips curve (12). We can then express the

Lagrangian of the program, denoted by $\mathcal{L}$. From this Lagrangian, we can define the (SVL) $\psi_{i,t}$ as:

$$\psi_{i,t} := \frac{\partial \mathcal{L}}{\partial c_{i,t}},$$

which is the value for the planner to transfer one extra unit of consumption good to agent i in period t. The expression of $\psi_{i,t}^{FP}$ is:

$$\psi_{i,t} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}} - \underbrace{\frac{\varepsilon_W - 1}{\psi_W}\gamma_{W,t}\frac{w_t y_{i,t} L_t}{1-\tau_t^E}u''(c_{i,t})}_{\text{effect on the bargained wage}} \tag{29}$$

As can be seen in equation (29), this valuation consists of three terms. The first is the marginal utility of consumption $\omega_t^i u'(c_{i,t})$, which is the private valuation of liquidity for agent i multiplied by the current weight of agent i. The second term in (29) takes into consideration the impact of the extra consumption unit on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$, and thus makes her Euler equation (either nominal or real) more "binding". This more "binding" constraint reduces the utility by the algebraic quantity $u''(c_{i,t})\lambda_{i,t}$. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity $\lambda_{i,t-1}$. The third term is the effect of a transfer to households on the marginal incentives to bargain the wage by the union.

This expression is common to all HA economies we consider, whatever the set of tools. The expressions of the first-order conditions of the planner will obviously depend on the set of tools. The provide all the derivations in Appendix ????.

The corresponding complete-market economy We solve the same problem with a representative agent instead of the heterogeneous-agent structure. We provide the problem and the first-order conditions in Appendix ????, to save some space, as the techniques to solve the model are more standard (although the problem as not been solved with a rich fiscal structure, to the best of our knowledge).

The MVPF

In both RA and HA economies, the MVPF is the Lagrange multiplier on the budget of the State. Although the steady-state value of the Lagrange multiplier will be the same in all HA economy (as the steady states are the same), its dynamics will depend on the state of tools S. As mentioned above, the $MVPF$ is now a very complex function of the allocation.

4.2 The equivalence result

We can state our main equivalence result.

Proposition 2 (An equivalence result) *In the HA economy, when all instruments* $S = (\tau_t^E, \tau_t^S, \tau_t^L, \tau_t^K)$ *are optimally chosen, the planner exactly implements* $\pi_t^P = 0$ *and* $\pi_t^W = 0$*, for both supply* (Z_t) *and demand shocks* (G_t, β_t).

Proposition 2 generalizes the equivalence result of Correia et al. (2008) and Correia et al. (2013) for representative agent economies and LeGrand et al. (2022) for heterogeneous-agent economy to the case where there are both sticky prices and sticky wages. Interestingly, compared to LeGrand et al. (2022), we need two additional instruments (τ_t^E, τ_t^S), whereas we introduce one additional nominal constraint. Indeed, we need one instrument to prevent wage inflation (which destroys resources) and another one to reproduce the flexible price labor supply and neutralize the market power of unions. In the presence of a sufficiently large fiscal system, monetary policy has no role but price stability. Importantly, the result requires the presence of two labor taxes. The first labor tax τ^S (internalized by the planner) enables the planner to "isolate" the pre-tax rate $\tilde{w}_t$ that is determined by the allocation (with a zero price inflation) from the union wage $\hat{w}_t$ that is determined by the inflation path $(\pi_t^W)_t$. Removing τ^S as an independent instrument imposes a constraint between the factor price $\tilde{w}_t$ and the wage inflation path. In other words, the planner would have to balance the effects of price inflation (determining $\tilde{w}_t$) and of wage inflation (determining $\hat{w}_t$). The second labor tax τ_t^E enables the planner to simultaneously set the labor supply optimally and close the wage gap in the wage Phillips curve. Removing τ_t^E would imply that the planner would need to tradeoff two inefficiencies: (i) the sub-optimal labor supply due the market power of unions and (ii) the cost of wage inflation. Should one of these two instruments be removed, Proposition 2 would not hold anymore and the economy would feature positive inflation on wages or on prices.

Overall, the first item of Proposition 2 rationalizes our tax system, which is the minimal tax system for which price stability is optimal.[8]

4.3 Theoretical results about inflation in HA to RA economies

The previous Proposition is the benchmark case. We can now consider a smaller set of time-varying instruments to analyze theoretically the dynamics and price stability for both supply and demand shock, in RA and HA economies. The Table 1 summarizes the results about the effect of missing instruments on price and wage inflation in both RA and HA. Note that these results only concern inflation but not the allocation. Inflation being close in HA and RA economies is

[8] More precisely, other tax systems could correspond to price and wage stability. For instance, it could be possible to consider time-varying consumption tax as in Correia et al. (2008). However, the number of independent instruments would not be smaller. We consider our tax system to be not unrealistic.

consistent with both allocation being very close or very different. In other words, the difference in inflation between HA and RA is not a sufficient statistics to assess the difference in the allocations of RA and HA economies. We show the difference in allocations between HA and RA economies below, as we have to simulate the economies (whereas the result of this Section are theoretically derived).

We focus on eight economies, to identify the main effects. We first focus on discount factor shocks β_t and public spending shocks (G_t). The first column provides the set of tools considered. The second, third and fourth column are the result for the RA economy for the β, G, Z shock respectively. The fifth column are the results for the HA economy for the three shocks altogether (β, G, Z). The first row is the benchmark economy with all instruments: the set of tools is $S^{(1)} = (\tau^K, \tau^L, \tau^S, \tau^E)$. The following six rows are providing result for different set of instruments, defined as missing instruments compared to $S^{(1)}$. For instance, $S^{(2)} = S^{(1)} - (\tau^S)$ is $S^{(2)} = \left(\tau^K, \tau^L, \tau^E\right)$.

First, in the RA economy a very limited set of tools is enough to reach the first-best allocation, and thus avoid price or wage inflation. Only when only distorting labor tax are available $S^{(10)} = \left(\tau^L\right)$, both price and wage inflation are used to finance public spending (G_t). The inflation tax is indeed used to raise resources and avoid to high distortions induced by the time-varying labor tax. For technology shocks, price and wage inflation change to reduce the difference between the real wage and the marginal productivity of labor.

In the HA economy, a time-varying τ_t^S is enough to implement price stability $\pi^P = 0$. When other instruments are missing, the planner has some incentives to deviate for price and wage stability to increase welfare.

To assess the deviation of the allocations and the quantitative deviation from price stability, we now provide a quantitative investigation of some relevant economies.

5 Quantitative analysis of optimal policies

We now focus on three economies to derive the main lessons for the difference between the HA and the RA allocations on the one hand, and for the deviation form price and wage stability on the other hand. Indeed, the simulation of the eight economies for the three shocks requires simulating 24 economies for both RA and HA environment. This is done in Appendix, but it is very tedious. The choice of the three simulations is done to identify the main mechanisms.

We consider first the economy with all fiscal instruments $S^{(1)} = (\tau^K, \tau^L, \tau^S, \tau^E)$ for β-shock. Second, we simulate the same economy for Z-shock. Third, we consider the economy the economy with time-varying labor tax only, $S^{(8)} = (\tau^L)$ for $Z-$shock and G- shock. We discuss in Section ???, the lessons for the other economies we simulate.

In order to provide these simulations we first calibrate the model in Section 5.1. We explain how to compute optimal policies in HA economies in Section 5.2.

<table>
<tr><th></th><th colspan="2">RA</th><th colspan="2">HA</th></tr>
<tr><th>Time-varying taxes</th><th>(β, G)</th><th>Z</th><th>(β, G)</th><th></th></tr>
<tr><td>$S^{(1)} = (\tau^K, \tau^L, \tau^S, \tau^E)$</td><td rowspan="9">$\pi^P = 0$ and $\pi^W = 0$</td><td rowspan="8">$\pi^P = 0$ and $\pi^W = 0$</td><td rowspan="2">$\pi^P = 0$ and $\pi^W = 0$</td><td rowspan="9">π</td></tr>
<tr><td>$S^{(2)} = S^{(1)} - (\tau^S)$</td></tr>
<tr><td>$S^{(3)} = S^{(1)} - (\tau^E)$</td><td>$\pi^P = 0$ and $\pi^W \neq 0$</td></tr>
<tr><td>$S^{(4)} = S^{(1)} - (\tau^L)$</td><td rowspan="6">$\pi^P \neq 0$ and $\pi^W \neq 0$</td></tr>
<tr><td>$S^{(5)} = S^{(1)} - (\tau^S, \tau^L)$</td></tr>
<tr><td>$S^{(6)} = S^{(1)} - (\tau^K)$</td></tr>
<tr><td>$S^{(7)} = S^{(1)} - (\tau^E, \tau^S)$</td></tr>
<tr><td>$S^{(8)} = S^{(1)} - (\tau^E, \tau^L)$</td></tr>
<tr><td>$S^{(9)} = S^{(1)} - (\tau^K, \tau^S, \tau^E)$</td><td>$\pi^P \neq 0$ and $\pi^W \neq 0$</td></tr>
</table>

Table 1: Price and wage inflation for different shocks (β, G, Z) and for different instruments, Representative Agent economy (RA) and Heterogeneous-agent economy (HA).

5.1 The calibration and steady-state distribution

Technology and TFP shock. The production function is: $Y = ZL$. The TFP process is a standard AR(1) process, with $Z_t = \exp(z_t)$ and $z_t = \rho_z z_{t-1}$, for $t \geq 1$, and $z_0 < 0$ is the period 0 negative TFP shock. We set $\rho_z = 0.95$, which the standard quarterly persistence.

Preferences. The period is a quarter. The discount factor is $\beta = 0.99$, and the period utility function is: $\frac{c^{1-\sigma}-1}{1-\sigma} - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in HA models. The scaling parameter is $\chi = 0.01$, which implies an aggregate labor supply of roughly 1/3.

The process for β_t is a AR(1) process, with $\beta_t = \beta * \exp(z_t)$ and $u_t = \rho_\beta u_{t-1}$, for $t \geq 1$, and $\beta_0 > \beta$ is the period 0 positive β-shock. We set $\rho_\beta = 0.95$ to consider the same persistence of all shocks.

Idiosyncratic risk. We use a standard productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate a persistence of the productivity process $\rho_y = 0.994$ and a standard deviation of $\sigma_y = 0.06$. These values are consistent with empirical estimates (Krueger et al., 2018), and generates a steady-state Gini of wealth of 0.78, which is again in line with the data.[9] Finally, we use the Rouwenhorst (1995) procedure to discretize the productivity process into 10 idiosyncratic states with a constant transition matrix.

[9]The Gini of wealth is 0.78 using the SCF data in 2007, before the 2008 Great Recession.

Steady state taxes and public debt, and public spending shock We first solve the model with constant exogenous taxes and explain below the choice of the Social Welfare Function (SWF). We first assume that employer social contributions and capital taxes are 0, $\tau^S = \tau^K = 0$. The income tax τ^E is set to $\frac{1}{\varepsilon_W}$ to undo distortions on the labor market due to the monopoly power of unions. We consider that $\tau^L = 16\%$. This last value (together with the value of public debt explained below) implies that public spending over GDP is 15, which is close to the US value in 2007. The amount of public debt (which is the only asset here) is set to the annual value of 1.28. As public debt is the only asset in our economy, we target this amount to obtain an average Marginal Propensity to Consume (MPC) of 0.3.[10]

The process for G_t is a AR(1) process, with $G_t = G * \exp(z_t)$ and $u_t = \rho_G u_{t-1}$, for $t \geq 1$, and $G_0 > G$ is the period 0 positive G-shock. We set again the same persistence.

Monetary parameters. Following the literature and in particular Schmitt-Grohé and Uribe (2005), we assume that the elasticity of substitution is $\varepsilon_P = 6$ across goods and $\varepsilon_W = 21$ across labor types. The price adjustment cost is set to $\psi_P = 100$, such that the slope of the price Phillips curve is $\frac{\varepsilon_P - 1}{\psi_P} = 5\%$ (see Bilbiie and Ragot, 2021, for a discussion and references). The wage adjustment cost is set to $\psi_W = 2100$, such that the slope of the wage Phillips curve is 1%, assuming wages to be stickier than prices.[11] Finally, as there is no inflation on prices or wages at the steady state: $\pi^P = \pi^W = 0$, these coefficients only matter in the dynamics.

Table 2 provides a summary of the model parameters.

Calibration of the complete-market economy. The calibration of the CM economy considers the same preference parameters as in the HA economy. We denote with upper-script CM (HA) the allocation in the CM (HA) economy. In the CM economy, the first-best can be achieved at the steady-state. The steady-state labor supply L^{CM} (with $\pi^W = 0$) is determined by $v'(L^{CM}) = u'(c^{CM})$. We set We set public spending (G^{CM}), in the CM economy such that public-spending-to-GDP are identical in the two economies: $G^{CM}/Y^{Cm} = G^{HA}/Y^{HA}$. The derivations of about the CM are provided in Appendix ???.

5.2 Simulating optimal policies in the HA economies

To investigate the optimal dynamics of the model, we recall that we perform the following experiment – which is standard in the New Keynesian RA literature, but which must be adapted to the HA case. We first solve for the optimal policy for a given set of instruments and consider

[10] We thus adopt a liquid one-asset liquid wealth calibration to match a realistic MPC (Kaplan and Violante, 2022).

[11] We have performed sensitivity analysis regarding these coefficients. Our qualitative results appear not to be sensitive to these values, even if inflation and wage volatility increases with the slopes of Phillips curves.

Parameter	Description	Value	Target
	Preference and technology		
β	Discount factor	0.99	Quarterly calibration
σ	Curvature utility	2	
$\bar{a}$	Credit limit	0	
χ	Scaling param. labor supply	0.01	$L = 1/3$
φ	Frisch elasticity labor supply	0.5	Chetty et al. (2011)
	Shock process		
ρ_y	Autocorrelation idio. income	0.993	Krueger et al. (2018)
σ_y	Standard dev. idio. income	6%	$Gini = 0.78$
ρ_z	Autocorrelation TFP shock	0.95	
	Tax system		
τ^L	Labor tax	16%	$G/Y = 15\%$
τ^E	Other tax	4.74%	$1/\varepsilon_w$
τ^S,τ^E,τ^K	Other tax	0%	
B/Y	Public debt over yearly GDP	128%	$MPC = 0.3$
G/Y	Public spending over yearly GDP	15%	Targeted
	Monetary parameters		
ε_p	Elasticity of sub. between goods	6	Schmitt-Grohé and Uribe (2005)
ψ_p	Price adjustment cost	100	Price PC 5%
ε_w	Elasticity of sub. labor inputs	21	Schmitt-Grohé and Uribe (2005)
ψ_w	Wage adjustment cost	2100	Wage PC 1%

Table 2: Parameter values in the baseline calibration. See text for descriptions and targets.

the steady-state allocation – which is the long run allocation in the absence of any aggregate shock. We then assume that the economy starts from the Ramsey steady-state and we then implement a period-0 transitory shock, which is either supply or demand driven. This procedure allows us to quantify how the economy is perturbated from that the steady state before converging back to the latter.

The steady state crucially depends on the Social Welfare Function used in the Ramsey program, as well as on the tools that the planner has access to. To overcome this difficulty and to start from the same steady state in all cases, we use the inverse optimal taxation approach, as in Heathcote and Tsujiyama (2021) and LeGrand and Ragot (2025). More precisely, we consider the same steady-state fiscal instruments, defined by $\tau^S = \tau^K = 0$, $\tau^E = 1/\varepsilon_w$ and $\tau^L > 0$, and estimate the weights of the SWF for each set of fiscal tools to ensure that this steady state is optimal. Indeed, each instrument of the planner generates a first-order condition, which imposes one restriction on the SWF.[12]We then choose the SWF satisfying these restrictions, which is the closest one to the utilitarian SWF (where all weights are equal). We also verify that the SWF does not quantitatively affect the dynamics of the allocation at the first order.

The Ramsey problem in HA models cannot be solved with simple simulation techniques. Indeed, the Ramsey equilibrium is now a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. While the steady-state values of Lagrange multipliers is already difficult to compute, the Ramsey solution actually requires the dynamics of this joint distribution. For this reason, we use the truncation method of LeGrand and Ragot (2022a) to determine the joint distribution of individual wealth and Lagrange multipliers.[13] The accuracy of optimal policies has been analyzed in LeGrand and Ragot (2023) for both the steady state and the dynamics. In addition, an improvement to efficiently reduce the state space is provided in LeGrand and Ragot (2022b). We detail the calculations in Appendix, and refer to these papers for details about the method.

To find the steady-state values of the Lagrange multipliers and SWF for a given fiscal policy, we use the following algorithm:

1. Set a truncation structure (a maximum truncation length N) and set instrument values.

2. Solve the steady-state allocation of the full-fledged Bewley model with the given instrument values, using standard techniques.

3. Consider the truncated representation of the economy, i.e., aggregate over truncated histories.

4. Compute the steady-state Ramsey solution in truncated economy

    (a) Derive first-order conditions of the planner for each instrument in the truncated representation.

    (b) Compute the SWF weights that are the closest to 1, for which all the planner's FOCs hold.

    (c) Compute associated Lagrange multipliers.

    (d) The truncated representation, together with the fiscal instruments, the estimated SWF, and Lagrange multipliers is a steady-state optimal Ramsey allocation for the truncated representation.

[12]This strategy ensures that a consistent steady-state exists. An alternative would be to consider a given SWF function and solve for the optimal Ramsey steady state. In this case, the steady-state allocation can be unrealistic (Auclert (2024) or LeGrand and Ragot (2025) for a discussion). As in standard New Keynesian models, optimal steady-state price and wage inflation is 0, whatever the social welfare function. As a consequence, steady-state price stability does not impose any restriction on the SWF.

[13]Optimizing on simple rules in the spirit of Krusell and Smith (1998) is also hard to implement as their are many independent instruments.

5. Compute the optimal dynamics of instruments and allocation in the truncated economy using the first order conditions of the planner – as is standard in any finite state space model.

We use the refined truncation approach, with a number of length for the refinement equals to $N = 8$. We check that the results do not depend on the choice of the truncation length. As in LeGrand and Ragot (2022a), the truncation provides accurate results, thanks to the introduction of the ξs parameters. We check that the dynamics does not depend on the truncation length.

5.3 The economy with all instruments and technology shocks (Z_t)

We provide results for the allocation and tools plotting the Impulse Response Functions (IRFs) after a transitory negative technology shock. We plot the path of nine key variables for 40 periods. We report the aggregate consumption (Panel 1) percentage proportional deviation (PPD) from the steady state (PPD), price inflation in percentage level deviation (PLD) from the steady state, the wage inflation in PLD, employer social contribution τ_t^S in PLD, the labor tax τ_t^L in PLD, the income tax in PLD, public debt in PPD, the MVPF in PPD, which is the Lagrange multiplier on the budget of the government, and the MVCC in PPD, which is the average normalized Lagrange multiplier on the credit constraints.

First, both price and wage inflations are zero in the dynamics as expected. Second, the aggregate allocation, is virtually identical between the HA and the CM economies. The change MVPF is nearly identical in the two economies (whereas the steady state values are different). Moreover, the MVCC barely moves in the HA economy and it is 0 in the CM economy by construction. The tools have a different dynamics in the HA and RA economies to implement the same allocation. In the CM economy, labor tax and employed social contributions are moving in the opposite direction for the pre-tax real wage to follow the productivity of labor, without generating moving in nominal bargained wage. In the HA economy, the same movement occurs and the income tax is used for the three taxes to impose the optimal level of redistribution, allowing the average MVCC to be almost constant, while keeping the bargained wage constant. Public debt decreases on impact as the economy is poorer and agents save less. These results are consistent with the results of the simple model of Section 2 : the HA and CM allocations are very close.

5.4 The economy with all instruments and discount factor shocks (β_t)

We provide results for the optimal allocation after positive discount factor shock in Figure 2, for the same variables as in Figure 1.

There is now a significant difference between the CM and HA economy for aggregate consumption. The path of the MVPF and the MVCC are now different. In the CM economy, the planner doesn't react to a discount factor shock, as the problem is static. In the HA economy,

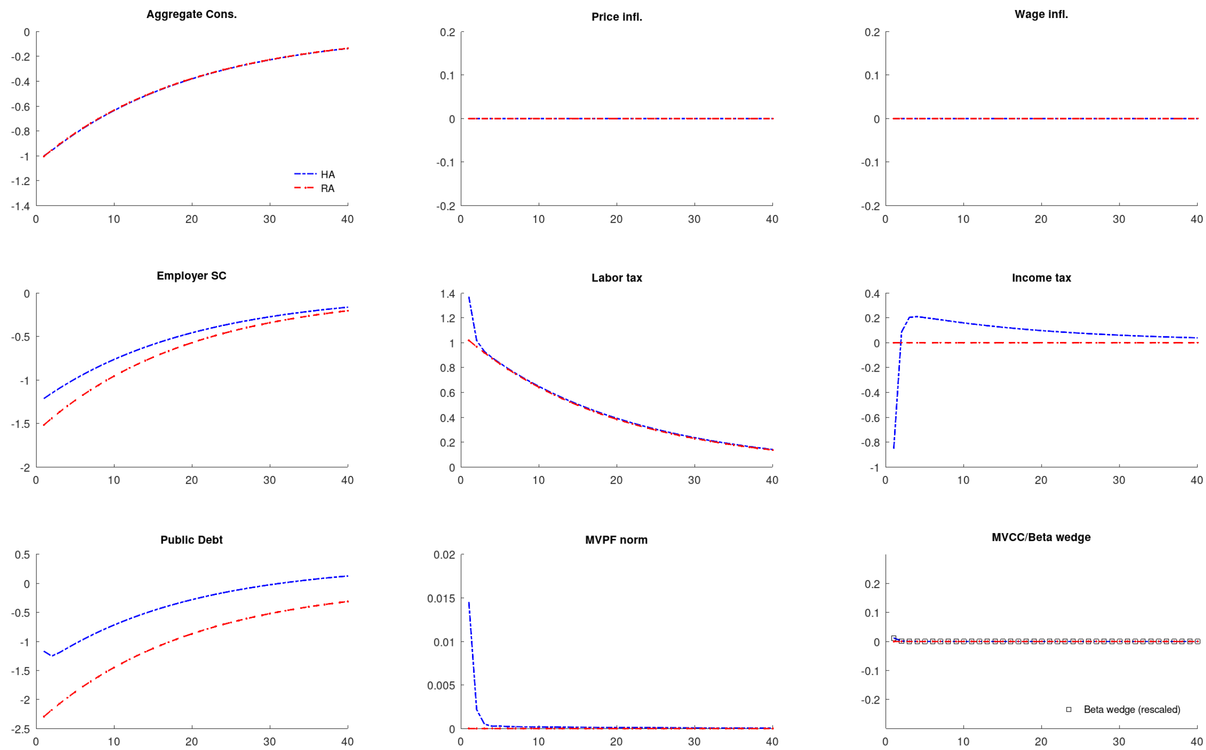

Figure 1: Dynamics of the economy for negative TFP shock for economy with all instruments $\left\{\tau_t^S, \tau_t^L, \tau_t^K, \tau_t^E\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Complete-Market (CM) is in red. Variables are in percentage proportional change, except tax rates and inflation rates which are in percentage level change.

price and wage inflation is 0 and the labor and income taxes are moving to implement a constant bargained wage. Public debt increases at impact as the households want to save more after such a shock. These results are again consistent with the ones of the simple model, the CM and HA models are different although inflation is the same in the two economies.

5.5 The economy with constant τ^L and τ^E after a public spending shock (G_t)

We provide results for the optimal allocation after a positive shock on public spending in Figure 3, for the same variables as in the previous figures. We now consider an economy where the labor tax and the income tax rates as set to their steady-state values.

First, one can observe that the allocation between CM and HA economies are different, as the MVPF and MVCC. Although wage inflation barely moves, price inflation increases on impact and it increases more in the HA economy than in the CM economy. This economy is an example of a case where the inflation is different.

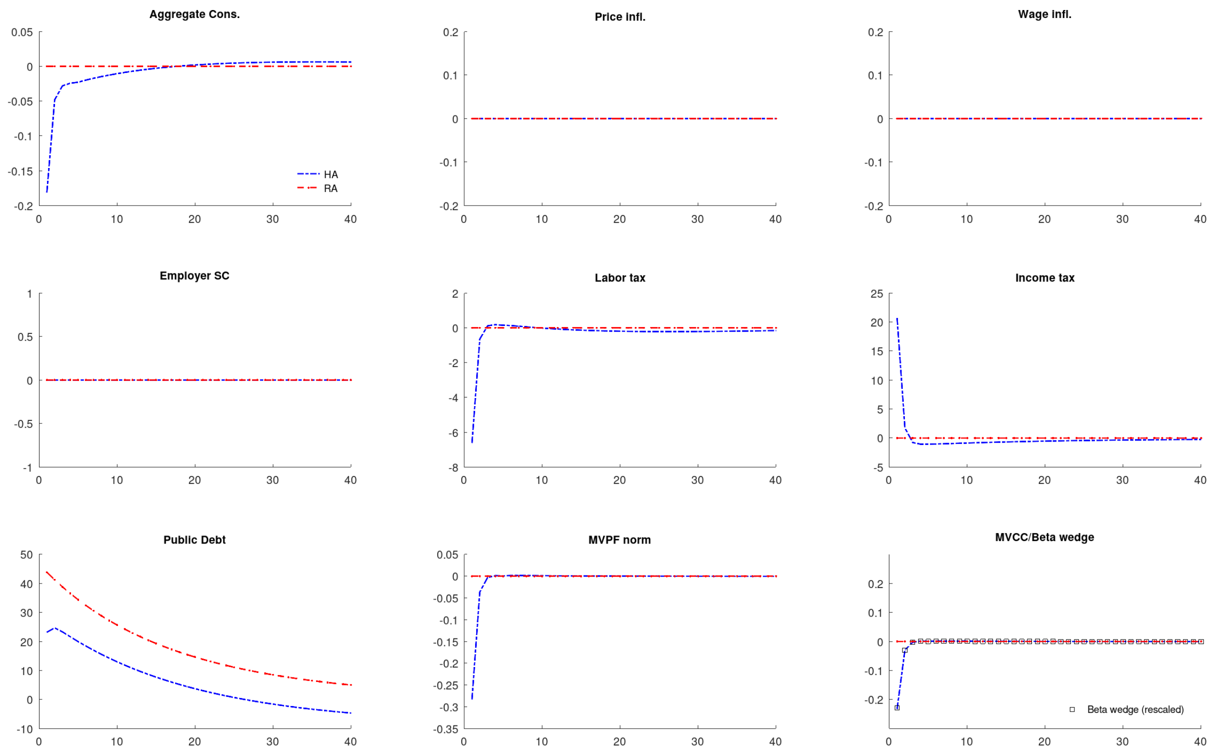

Figure 2: Dynamics of the economy for negative TFP shock for economy with all instruments $\left\{\tau_t^S, \tau_t^L, \tau_t^K, \tau_t^E\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Complete-Market (CM) is in red. Variables are in percentage proportional change, except tax rates and inflation rates which are in percentage level change.

5.6 The effect of missing instruments

We now provide the result with additional simulations. To save some space, we summarize results and refer to the IRFs in Appendix.

- First, we considered the economy keeping the capital tax constant, and considering $\left\{\tau_t^S, \tau_t^L, \tau_t^E\right\}$. For the given calibration, inflation barely moves on impact. When we decrease the coefficient on price stickiness to a low value of $\psi_p = 10$ instead of 100, then inflation increases significantly on impact, for one period. This spike in inflation is a substitute for the missing capital tax. It indeed decreases the real interest rate for one period due to the Fisher effect, as it is unexpected. As a consequence, inflation is a substitute for missing capital tax, when price are flexible enough, this result is consistent with LeGrand et al. (2022).
- Second, we find that the allocations in HA and CM models after positive public spending shocks are similar to negative TFP shocks, as mentioned above. The difference between HA and CM allocation and price increases when there the number of instruments decreases

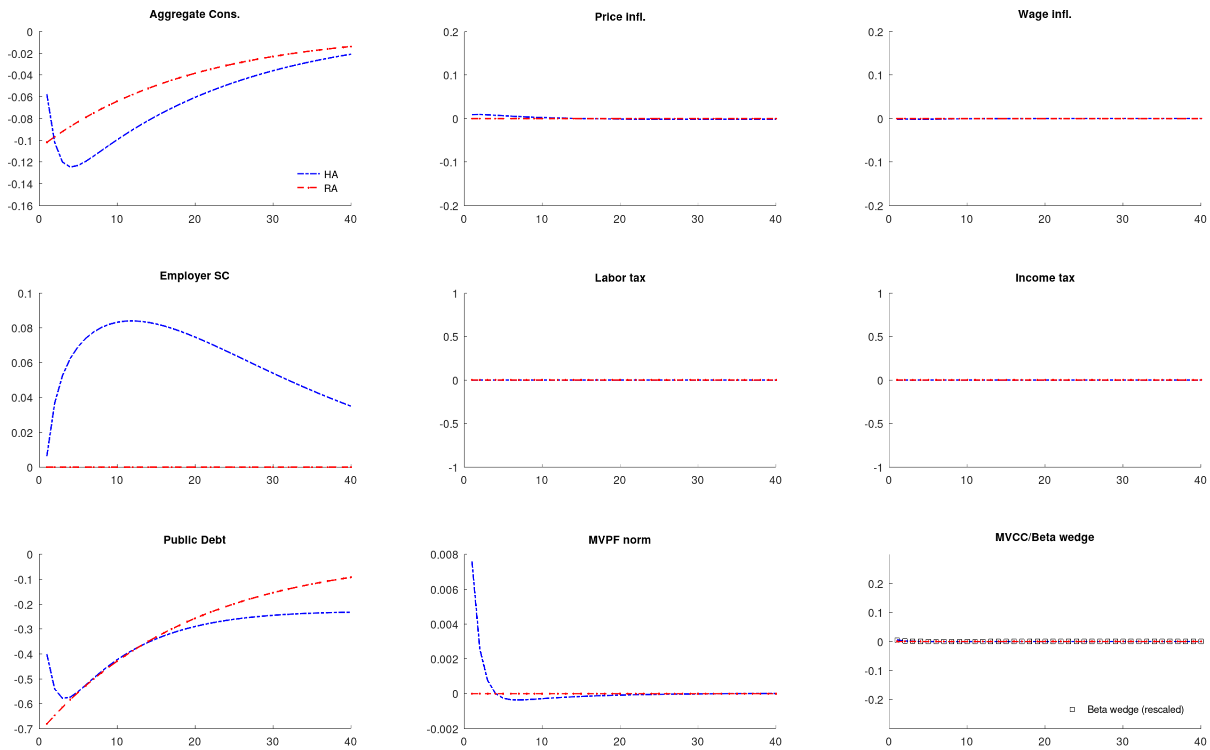

Figure 3: Dynamics of the economy for negative TFP shock for economy with instruments $\left\{\tau_t^L, \tau_t^K\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Complete-Market (CM) is in red. Variables are in percentage proportional change, except tax rates and inflation rates which are in percentage level change.

- Third, the discount factor shock is generating the biggest difference between HA and CM economies, as the MVCC varies significantly in HA economies.

- Fourth, for public spending shock and discount factor shock, the use of time-varying employers' social contribution (which is a time-varying labor subsidy or taxes not directly affecting the bargaining of wages) is sufficient tool to quantitatively reduce optimal inflation variability.

- Finally, the path of public debt varies significantly for HA and CM economies, even when allocations are close. Indeed, in HA economy, the planner adjust to the demand for liquidity for agents to self-insure.

6 The welfare effect of missing instruments

TO BE DONE

7 Conclusion

We have solved for optimal fiscal and monetary policy in an economy plagued with both sticky prices and sticky wages, considering various fiscal instruments. We compare the outcome of a complete-market economy and an heterogeneous-agent economy for three shocks, a TFP shock, a public spending shock and discount factor shock. As a benchmark, we first provide a complete fiscal system, which is defined as a fiscal system for which both price and wage inflation are zero after the three shocks. In other words, for the fiscal system any possibly useful effects of inflation are more efficiently captured by time-varying taxes, which avoid additional inflation costs.These complete fiscal system is a theoretical benchmark that the we use to consider both allocation and inflation when fixed some fiscal tools in the dynamics.

When all instruments are available the allocation of HA and CM economies are very close for both TFP and public spending shock, but very different for discount factor shock. This result comes from the different effect of each shock on the average tightness of the credit constraint of agents, that we have labelled the Marginal Value of Credit Constraint (MVCC). This result helps rationalize different outcome found in the literature, which appears to be shock specific.

We then explore the effect of missing fiscal tools on optimal inflation. In particular, we identify the fiscal tools which are necessary to implement price stability. This allows to understand the wedges that inflation is affecting to improve welfare, and provide some insight for monetary policy with heterogeneous agents. We find that inflation is a substitute for missing capital taxes if prices are flexible enough. Long-lasting inflation is used to reduce the labor wedge, which is the difference between the real wage rate and the marginal productivity of labor, when some labor taxes are not-time-varying.

Finally, we analyze the welfare effect of missing fiscal tools. This last experiment is interesting to identify the most welfare-enhancing fiscal tool after macroeconomic shocks. There is a consensus that monetary is the relevant instrument for business cycle stabilization, except at the zero lower bound. It is interesting to observe that it is not necessarily the case with nominal wage rigidity. Fiscal instrument can significantly improve welfare, but they are shock specific. In particular, time-varying labor subsidy is a useful tools to reduce the labor wedge after a TFP shock. It is noteworthy that these tools have been recently been used in Europe to stabilize employment. In Germany, the so-called *kurzarbeit* device played this role, while in France, the *activité partielle* policy was a wage subsidy to reduce layoffs during the Covid-19 crisis. These tools time-varying fiscal policy non-Keynesian stabilizers are their goal is not to simply manage aggregate demand and economic activity due to fiscal multipliers, but to directly affect the labor wedge.

From this analysis, we conclude that HA economies provide new insight about optimal stabilization policy, which appear to be shock specific.

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Appendix

A The Simple model

To be written

B Derivation of the wage-Phillips curve

Recall that the objective of union k is thus:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2} \left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1 \right)^2 \right) \ell(di),$$

subject to (**??**) and where $c_{i,t}$ and $l_{i,t}$ are the consumption and labor supply of agent i. The first-order condition with respect to W_{kt} thus writes as:

$$\pi_t^W(\pi_t^W + 1) = \frac{\hat{W}_{kt}}{\psi_W} \int_i \left(u'(c_{i,t}) \frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} - v'(l_{i,t}) \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \right) \ell(di) + \beta \mathbb{E}_t \left[\pi_{t+1}^W(\pi_{t+1}^W + 1) \right], \tag{30}$$

where the wage inflation rate is denoted by:

$$\pi_t^W = \frac{\hat{W}_{k,t}}{\hat{W}_{k,t-1}} - 1.$$

The labor supply l_{it} of agent i is the sum of her hours l_{ikt} supplied to union k, summed over all unions: $l_{it} = \int_k l_{ikt} dk$. Each union is assumed to request its members to supply an uniform number of hours, such that: $l_{ikt} = L_{kt}$. We thus deduce from (**??**):

$$\hat{W}_{kt} \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} = \hat{W}_{kt} \frac{\partial \left(\int_k \left(\frac{\hat{W}_{kt}}{\hat{W}_t} \right)^{-\varepsilon_W} L_t dk \right)}{\partial \hat{W}_{kt}} = -\varepsilon_W L_{kt}. \tag{31}$$

To compute the derivative of consumption $\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}}$, it should observed that it is equal to the derivative of its net total income. The net total income of agent i writes as $(1-\tau_t^L)\hat{W}_{kt} y_{i,t} l_{i,t}/P_t$, where τ_t^L is the labor tax. Formally:

$$\begin{aligned}
\frac{1}{c_{i,t}} \frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= \frac{1}{\hat{W}_{kt}} + \frac{1}{l_{i,t}} \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \\
&= \frac{1}{\hat{W}_{kt}} - \frac{\varepsilon_W}{\hat{W}_{kt}} \frac{L_{kt}}{l_{i,t}} \\
\hat{W}_{kt} \frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= (1-\varepsilon_W)(1-\tau_t^L) \hat{W}_{kt} y_{i,t} l_{i,t}/P_t
\end{aligned} \tag{32}$$

We focus on the symmetric equilibrium where all unions choose to set the same wage $\hat{W}_{kt} = \hat{W}_t$, hence all households work the same number of hours, equal to $l_{it} = L_t$. Combining (30) with the partial derivatives (31) and (32), we deduce the following Phillips curve for wage inflation:

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1 - \tau_t^L)\hat{w}_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}} L_t + \beta \mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W + 1)\right], \tag{33}$$

where $\hat{w}_t = \hat{W}_t / P_t$ is the real pre-tax wage.

C Proof of proposition ??

We solve for the Ramsey allocation in the RA case, for both demand and supply shocks.

C.1 First-best allocation

In the first-best allocation, the resource constraint imposes that total consumption is financed out of production: $G_t + C_t = Z_t L_t$. The labor supply is thus determined by the solution to the following program: $\max_{L_t} u(Z_t L_t - G_t) - v(L_t)$. The first-order condition defines the first-best labor supply L_t^{FB} as the solution of:

$$Z_t u'(Z_t L_t^{FB} - G_t) \quad = \quad v'(L_t^{FB}), \tag{34}$$

which can be shown to admit a unique solution under standard assumption (u increasing concave with $u'(0) = \infty$ and $u'(\infty) = 0$ and v increasing convex).

Consider the following particular case. We set $G_t = 0$, $u'(c) = c^{-\gamma}$, and $v'(L) = \chi^{-1} L^{1/\phi}$ such that $\gamma > 0$ is the inverse of the IES and $\phi > 0$ is the Frisch elasticity of labor supply. We obtain: $L_t^{FB} = \chi^{\frac{1}{\frac{1}{\phi}+\gamma}} Z_t^{\frac{1-\gamma}{\frac{1}{\phi}+\gamma}}$.

C.2 Representative-agent model with a full set of instruments

We show that when the planner has access to the full set of instrument, the first-best allocation can be implemented for both demand and supply shocks. This requires $\pi^W = \pi^P = 0$, to avoid price or wage adjustment costs. The equations defining the equilibrium allocation are

$$w_t = \left(1 - \tau_t^E\right)(1 - \tau_t^L)\left(1 - \tau_t^S\right) Z_t,$$

$$v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1-\tau_t^E} u'(C_t) = 0 \tag{35}$$

$$\frac{1}{Z_t} \frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)} = 1 \tag{36}$$

$$1 = \frac{w_t}{w_{t-1}} \frac{(1-\tau_{t-1}^L)}{(1-\tau_t^L)} \tag{37}$$

$$C_t = Z_t L_t - G_t$$

$$G_t + (1+r_t) B_{t-1} + (w_t - Z_t) L_t = B_t \tag{38}$$

$$u'(C_t) = \beta (1 + r_{t+1}) u'(C_{t+1}) \tag{39}$$

Note that the Euler equation (39) determines the real interest r_t^{FB} $t \geq 1$ from the first-best path of consumption C_t^{FB}. Importantly, this equations don't determine the period-0 interest rate r_0.

Equation (38à is the budget constraint of the government.

Equation (37) implies that there is a α such that $1 - \tau_t^L := \alpha w_t$. For the allocation to be the first best, equations (35) and (34) implies that

$$1 - \tau_t^E \quad := \quad \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{Z_t}$$

Then equation (36) implies

$$1 - \tau_t^S := \frac{\varepsilon_W}{\varepsilon_W - 1} \frac{1}{\alpha w_t}$$

Then the budget of the government implies

$$w_t = Z_t - \frac{G_t + (1+r_t) B_{t-1} - B_t}{L_t} \tag{40}$$

Implementation results: For any path of G_t, Z_t and path of public debt B_t, for $t \geq 0$, the first best can be implemented.

The proof is direct. Consider a path G_t, Z_t, B_t and the first-best labor supply L_t^{FB}. It gives a path of consumption determining the real interest rate r_t, $t \geq 1$. For any r_0 (which is an additional free variable), the equation (40) determines a path for the real wage rate. Then for any α, $1 - \tau_t^L = \alpha w_t$, $1 - \tau_t^E = \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{Z_t}$, $1 - \tau_t^S = \frac{\varepsilon_W}{\varepsilon_W - 1} \frac{1}{\alpha w_t}$ is a market equilibrium.

Note thus that public debt is not determined in this implementation.

Note that in a steady-state equilibrium (where $Z = 1$ and B, G, w are constant) , we have

$$B_{SS} = \frac{\beta}{1-\beta} \left((1-w) L_{SS}^{FB} - G_{SS} \right)$$

C.3 RA model : Representative agents without time-varying τ_t^E

We assume that the economy is in steady state, where public debt is B_{SS}. and hit by the shock at period 0. In the previous analysis, we can impose $\tau_t^E = \tau_{SS}^E$.

$$w_t = \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1 - \tau_{ss}^E\right) Z_t$$

Then, $1 - \tau_t^L = \alpha \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1 - \tau_{ss}^E\right) Z_t$, , $1 - \tau_t^S = \frac{1}{\alpha(1-\tau_{ss}^E)Z_t}$.

The budget of the state implies (for $t \geq 0$, with the notation $B_{-1} = B_{SS}$)

$$(1 + r_t) B_{t-1} - B_t = \Theta_t$$

with

$$\Theta_t := \left(1 - \frac{\varepsilon_W}{\varepsilon_W - 1}\left(1 - \tau_{ss}^E\right)\right) Z_t L_t^{FB} - G_t$$

The variable Θ_t is uniquely determined. This uniquely determines the path of public converging back to the steady state. To see that, first observe that the period-0 interest rate 0 r_0 is a free parameter determined by period-0 capital tax $\hat{\tau}_0^K$.

$$B_{-1}(1 + r_0) = \sum_{t=0} \frac{G_t}{R_{0,t}} + \lim_{T\to\infty} \frac{B_T}{R_{t,T}}$$

To have $\lim_{T\to\infty} \frac{B_T}{R_{t,T}} = 0$, we must choose the initial capital tax such that

$$(1 + r_0) B_{-1} = \sum_{k=t}^{\infty} \frac{\Theta_k}{\prod_{j=t+1}^{k}\left(1 + r_j^{FB}\right)} + \lim_{T\to\infty} \frac{B_T}{\prod_{j=t+1}^{T}\left(1 + r_j^{FB}\right)}$$

(with the notation $\prod_{j=t+1}^{t} = 1$). The term $\lim_{T\to\infty} \frac{B_T}{\prod_{j=t+1}^{T}(1+r_j)} = 0$ if the economy converges back to the steady state. The unique period 0 allowing the public debt to converge back to the steady state is

$$1 + r_0 = \frac{\sum_{k=t}^{\infty} \frac{\Theta_k}{\prod_{j=t+1}^{k}(1+r_j^{FB})}}{B_{SS}},$$

which is uniquely determined.

C.4 RA with Demand shock

We now show that whatever the fiscal system (economy 3 and 4), the first-best allocation can be implemented with demand shocks. The proof follows the consideration of the previous Section.

We now assume that $\tau^E = \tau_{SS}^E$ and $\tau^E = \tau_{SS}^E$. We now focus on the case where $1 - \tau_{ss}^E = \frac{\varepsilon_W - 1}{\varepsilon_W}$, to determine uniquely the path of the instruments. Any other value would not quantitatively change the allocation, and qualitatively the path of the instruments.

In this case, we have $w = Z = 1$. Then $1 - \tau_t^L = \alpha$, and $1 - \tau^S = \frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{\alpha}$ and

$$1 + r_0 = \frac{1-\beta}{\beta}\sum_{k=t}^{\infty}\frac{G_k/G_{SS}}{\prod_{j=t+1}^{k}\left(1 + r_j^{FB}\right)},$$

implements the first-best allocation.

C.5 The RA economy without time-varying τ_t^E and τ_t^S, with optimal τ_t^L and supply shocks

The first-best cannot be implemented, and we must solve for the Ramsey allocation. We provide equations for both demand and supply shocks and then discuss each case in turn.

The program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,\Omega_t,\tilde{R}_t^N,L_t,c_t,a_t\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t) - v(L_t)\right) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t)B_{t-1} + w_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t L_t + B_t,$$

$$c_t + a_t = (1+r_t)B_{t-1} + w_t L_t,$$

$$u'(c_t) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{t+1})\Big],$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1-\tau_{ss}^E}u'(c_t)\right)L_t + \beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_{ss}^E)} - 1\right) + \beta\mathbb{E}_t\left(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} = \frac{w_t}{1-\tau_t^L}(1+\pi_t^P),$$

Define

$$T_t = (1+r_t)B_{t-1} - B_t$$
$$x_t = \frac{w_t}{1-\tau_t^L}$$

Then the program is (using $\frac{\varepsilon_W - 1}{\varepsilon_W} = 1 - \tau^E_{ss}$)

$$
\begin{aligned}
&\max_{\left(x_t T_t, \pi^P_t, \pi^W_t, w_t, L_t, c_t\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - v(L_t)\right) - \frac{\psi_W}{2}(\pi^W_t)^2\right], \\
&G_t + T_t + \left(1 - \tau^L_t\right) x_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi^P_t)^2\right) Z_t L_t, \\
&c_t = T_t + \left(1 - \tau^L_t\right) x_t L_t, \\
&\pi^W_t(\pi^W_t + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \left(1 - \tau^L_t\right) x_t u'(c_t)\right) L_t + \beta \mathbb{E}_t\left[\pi^W_{t+1}(\pi^W_{t+1} + 1)\right], \\
&\pi^P_t(1 + \pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}\frac{1}{Z_t} x_t - 1\right) + \beta \mathbb{E}_t\left(\pi^P_{t+1}(1 + \pi^P_{t+1})\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right), \\
&(1 + \pi^W_t) x_{t-1} = x_t(1 + \pi^P_t),
\end{aligned}
$$

while the corresponding Lagrangian becomes $c_t = T_t + \left(1 - \tau^L_t\right) x_t L_t$

$$
\begin{aligned}
\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(L_t) - \frac{\psi_W}{2}(\pi^W_t)^2) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1}) \pi^W_t (1 + \pi^W_t) \\
&+ \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \left(1 - \tau^L_t\right) x_t u'(c_t)\right) L_t \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1}) \pi^P_t (1 + \pi^P_t) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{\varepsilon_W}{\varepsilon_W - 1} x_t - Z_t\right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left((1 - \frac{\psi_P}{2}(\pi^P_t)^2) Z_t L_t - G_t - T_t - \left(1 - \tau^L_t\right) x_t L_t\right) \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1 + \pi^W_t) x_{t-1} - x_t (1 + \pi^P_t)\right)
\end{aligned}
$$

We now turn to the computation of the FOCs.

Consider

$$
\psi_t := \frac{d\mathcal{L}}{dc} = u'(c_t) - \underbrace{\frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(1 - \tau^L_t\right) x_t L_t u''(c_t)}_{\text{effect on wage inflation}}
$$

FOC wrt π^W_t.

$$
-\psi_W \pi^W_t - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi^W_t + 1) + \Lambda_t x_{t-1} = 0.
$$

FOC wrt π^P_t.

$$
-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi^P_t + 1) - \mu_t \psi_P \pi^P_t - \frac{\Lambda_t}{Z_t L_t} x_t = 0.
$$

FOC wrt x_t.

$$0 = \left(1-\tau_t^L\right) L_t \psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(1-\tau_t^L\right) u'(c_t) L_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1} L_t - \mu_t \left(1-\tau_t^L\right) L_t - \Lambda_t (1+\pi_t^P) + \beta\Lambda_{t+1}($$

FOC wrt L_t.

$$0 = \left(1-\tau_t^L\right) x_t \psi_t - v'(L_t) + \mu_t \left(\left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t - \left(1-\tau_t^L\right) x_t\right) + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - \left(1-\tau_t^L\right) x\right.$$
$$- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1} x_t - Z_t\right).$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^L$.

$$0 = L_t x_t \psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t} x_t u'(c_t) L_t - \mu_t x_t L_t.$$

Simplifying

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^L$.

$$0 = \psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t} u'(c_t) - \mu_t.$$
$$0 = \frac{\varepsilon_W}{\psi_W}\gamma_{W,t} u'(c_t)\left(1 - \frac{-\left(c_t - T_t\right) u''(c_t)}{u'(c_t)}\right)$$

In this case, one can check that one has $\gamma_{W,t} = 0$ (The wage Phillips curve is not a constraint)

FOC wrt π_t^W.

$$-\psi_W \pi_t^W + \Lambda_t x_{t-1} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} x_t = 0.$$

FOC wrt x_t.

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1} L_t - \Lambda_t (1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

Using the FOC wrt to τ^L, we have:

FOC wrt L_t.

$$\begin{aligned} 0 = &-v'(L_t) + \mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t \\ &- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}x_t - Z_t\right). \end{aligned}$$

Simplifying

$$\psi_W \pi_t^W = \Lambda_t x_{t-1}.$$

and

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) = \mu_t \psi_P \pi_t^P + \frac{\Lambda_t}{Z_t L_t} x_t.$$

FOC wrt x_t.

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1}L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$\begin{aligned} v'(L_t) = &\mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t \\ &- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}x_t - Z_t\right). \end{aligned}$$

Determining the path of public debt from the path of T_t

The dynamics of public debt is

$$B_t = (1 + r_t)B_{t-1} + T_t$$

At the moment of the shock, at period 0, the planner can change capital tax.

$$1 + r_0 = \frac{-\sum_{k=t}^{\infty} \frac{T_k}{\prod_{j=t+1}^{k}(1+r_j^{FB})}}{B_{SS}},$$

C.6 RA analysis without (time-varying) τ_t^E, τ_t^L, with time-varying τ_t^S

With the same change of variable as in the previous case,

$$
\max_{\left(\tau_t^L,\tau_t^S,\tau_t^E,\tau_t^K,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right)\ell(di)-\frac{\psi_W}{2}(\pi_t^W)^2\right],
$$

$$
G_t+T+w_tL_t\leq\Big(1-\frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_tL_t,
$$

$$
c_t=T_t+w_tL_t,
$$

$$
\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\Big(v'(L_t)-w_tu'(c_t)\ell(di)\Big)L_t+\beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],
$$

$$
\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}(\frac{1}{Z_t}\frac{w_t}{(1-\tau_{ss}^L)(1-\tau_t^S)(1-\tau_{ss}^E)}-1)+\beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\Big),
$$

$$
(1+\pi_t^W)w_{t-1}=w_t(1+\pi_t^P).
$$

Define

$$
z_t=\frac{1}{(1-\tau_{ss}^L)(1-\tau_t^S)(1-\tau_{ss}^E)}=\frac{1-\tau_{ss}^S}{1-\tau_t^S}
$$

$$
\begin{aligned}
\mathcal{L}=&\ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(u(c_t)-v(L_t)-\frac{\psi_W}{2}(\pi_t^W)^2)\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{W,t}-\gamma_{W,t-1})\pi_t^W(1+\pi_t^W)\\
&+\frac{\varepsilon_W}{\psi_W}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{W,t}\left(v'(L_t)-w_tu'(c_t)\right)L_t\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{P,t}-\gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_tL_t+\frac{\varepsilon_P-1}{\psi_P}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{P,t}\left(wz_t-Z_t\right)L_t\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\Big((1-\frac{\psi_P}{2}(\pi_t^P)^2)Z_tL_t-G_t-T_t-w_tL_t\Big)\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Lambda_t\left((1+\pi_t^W)w_{t-1}-w_t(1+\pi_t^P)\right)
\end{aligned}
$$

FOC wrt π_t^W.

$$
-\psi_W\pi_t^W-(\gamma_{W,t}-\gamma_{W,t-1})(2\pi_t^W+1)+\Lambda_tw_{t-1}=0.
$$

FOC wrt π_t^P.

$$
-(\gamma_{P,t}-\gamma_{P,t-1})(2\pi_t^P+1)-\mu_t\psi_P\pi_t^P-\frac{\Lambda_t}{Z_tL_t}w_t=0.
$$

FOC wrt w_t.

$$
0=L_t\psi_t-\frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)L_t+\frac{\varepsilon_P-1}{\psi_P}\gamma_{P,t}L_t-\mu_tL_t-\Lambda_t(1+\pi_t^P)+\beta\Lambda_{t+1}(1+\pi_{t+1}^W).
$$

FOC wrt L_t.

$$0 = w_t\psi_t - v'(L_t) + \mu_t\left(\left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t - w_t\right) + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - w_t u'(c_t)\right)$$
$$- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(wz_t - Z_t\right).$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt z_t.

$$0 = \gamma_{P,t}.$$

Simplifying

FOC wrt π_t^W.

$$\psi_W\pi_t^W = -(\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t w_{t-1}.$$

FOC wrt π_t^P.

$$-\mu_t\psi_P\pi_t^P = \frac{\Lambda_t}{Z_t L_t}w_{t-1}.$$

FOC wrt w_t.

$$0 = -\frac{\varepsilon_W}{\psi_W}\gamma_{W,t}u'(c_t)L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$v'(L_t) = +\mu_t\left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - w_t u'(c_t)\right)$$

$$\mu_t = u'(c_t).$$

D Ramsey program for HA models

D.1 Flexible-price equilibrium

We here assume here that the planner must choose a common labor supply for all agents, in a flexible price economy: $\pi_t^P = \pi_t^W = 0$. The program is:

$$\max_{\left(\tau_t^L, \tau_t^S, \tau_t^K, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) \right],$$

$$G_t + (1+r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t} \ell(di),$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1+r_t) a_{i,t-1} + w_t y_{i,t} L_t,$$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1+r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t}.$$

$$u'(c_{i,t}) = \frac{\beta}{1 - MVCC_{i,t}} \mathbb{E}_t \Big[(1+r_{t+1})\, u'(c_{i,t+1}) \Big]$$

The Lagrangian can be written as:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t)) \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,c,t} - (1+r_t) \lambda_{i,c,t-1} \right) u'(c_{i,t}) \ell(di)$$
$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(Z_t L_t + \int_i a_{i,t} \ell(di) - G_t - (1+r_t) \int_i a_{i,t-1} \ell(di) - w_t L_t - T_t \right).$$

We recall that $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - \left(\lambda_{i,c,t} - (1+r_t) \lambda_{i,c,t-1} \right) u''(c_{i,t})$. Compared to (29), we drop the FP subscript for the sake of simplicity. We compute the FOCs wrt four independent instruments: r_t, w_t, L_t and $(a_{i,t})_i$. The other instruments can be recovered from the constraints.

FOC wrt r_t.

$$\int_i a_{i,t-1} \hat{\psi}_{i,t} \ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t}) \ell(di) = 0. \tag{41}$$

FOC wrt w_t.

$$\int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) = 0.$$

FOC wrt L_t. Using the FOC on w_t:

$$\int_i \omega_{i,t} \ell(di) v'(L_t) = \mu_t Z_t = Z_t \int_i y_{i,t} \psi_{i,t} \ell(di).$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

D.2 The HA economy with all instruments

The program is:

$$\max_{\left(\tau_t^L, \tau_t^S, \tau_t^E, \tau_t^K, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t + \int_i a_{i,t} \ell(di),$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t,$$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) u'(c_{i,t+1}) \right] + \nu_{i,t},$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1 - \tau_t^E} \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right],$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^S)(1 - \tau_t^E)} - 1 \right) + \beta \mathbb{E}_t \left(\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \right),$$

$$(1 + \pi_t^W) \frac{w_{t-1}}{1 - \tau_{t-1}^L} = \frac{w_t}{1 - \tau_t^L} (1 + \pi_t^P).$$

We can set:

- τ_t^S such that $1 - \tau_t^S = \frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^E)}$, hence $\frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^S)(1 - \tau_t^E)} - 1$ and $\pi_t^P = 0$.
- τ_t^E is a free parameter that can be deduced from π_t^W and the allocation. Hence, the wage Phillips curve is not a constraint.
- π_t^W only reduces utility and is an independent parameter that can be set through τ^L, hence $\pi_t^W = 0$

The program then reduces to the same one as in the flexible-price economy without union:

Recovering taxes from the allocation We then have

$$1 - \tau_t^E = \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \frac{\int_i y_{i,t} u'(c_{i,t}) \ell(di)}{v'(L_t)}$$

$$1 - \tau_t^L = \alpha w_t$$

$$1 - \tau_t^S = \frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^E)}$$

D.3 The HA economy without τ_t^E

We impose $\tau_t^E = 0$. The program is otherwise the same as in Section D.2. In particular, τ_t^S only appears in the price Phillips curve. As consequence, this equation is not a constraint and τ_t^S is set, such that $\pi_t^P = 0$. Inflation indeed only destroys resources here. We then obtain the following program:

$$\max_{\left(\tau_t^L, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t)\right) \ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t) \int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t}\ell(di),$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + y_{i,t} w_t L_t + T_t,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t + \beta \mathbb{E}_t \Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],$$

Because of $\tau_t^E = 0$, we cannot have simultaneously optimal labor supply and $\pi_t^W = 0$: the planner has to balance the relative costs of wage inflation with the suboptimal provision of labor supply. The Lagrangian is:

$$\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W(1+\pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t) \int_i a_{i,t-1}\ell(di) - w_t L_t - T_t\right).
\end{aligned}$$

We recall that in this economy, we have $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u''(c_{i,t}) - \frac{\varepsilon_W - 1}{\psi_W} \gamma_{W,t} w_t y_{i,t} u''(c_{i,t}) L_t$, where we also drop the superscript.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = \gamma_{W,t}\frac{\varepsilon_W - 1}{\psi_W}\int_i y_{i,t}u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$-\int_i \omega_{i,t}\ell(di)v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) = 0.$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta\mathbb{E}_t\left[(1+r_{t+1})\hat{\psi}_{i,t+1}\right].$$

D.4 The HA economy without τ_t^E and τ_t^S with τ_t^L

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,\Omega_t,\tilde{R}_t^N,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i) - v(L_t)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\bar{a}, \nu_{i,t}(a_{i,t} + \bar{a}) = 0,\ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\left[(1+r_{t+1})u'(c_{i,t+1})\right] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}w_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W + 1)\right],$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)} - 1\right) + \beta\mathbb{E}_t\left(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} = \frac{w_t}{1-\tau_t^L}(1+\pi_t^P),$$

while the corresponding Lagrangian becomes:

$$
\begin{aligned}
\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_t^L)} - Z_t \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big) \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} - \frac{w_t}{1-\tau_t^L}(1+\pi_t^P) \right)
\end{aligned}
$$

We now turn to the computation of the FOCs.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^L} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t}\frac{w_t}{1-\tau_t^L} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t. Using the FOC wrt to τ^L, we have:

$$0 = \int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W - 1}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$
\begin{aligned}
0 = &-\int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) \\
&- (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{w_t}{(1-\tau_t^L)} - Z_t\right).
\end{aligned}
$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

FOC wrt τ_t^L. We derive wrt $\frac{1}{1-\tau_t^L}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} L_t - \Lambda_t (1 + \pi_t^P) + \beta \mathbb{E}_t \left[\Lambda_{t+1} (1 + \pi_{t+1}^W) \right].$$

D.5 The HA economy without time-varying τ_t^E and τ_t^L with τ_t^S

I consider $\tau_{ss}^E = \left(\frac{\varepsilon_W - 1}{\varepsilon_W} \right)^{-1}$ and $\tau_t^L = \tau_{ss}^L$

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^S, \tau_t^K, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, \Omega_t, \tilde{R}_t^N, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t + T_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t + \int_i a_{i,t} \ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t},$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t \Big[\pi_{t+1}^W (\pi_{t+1}^W + 1) \Big],$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1 - \tau^L)(1 - \tau_t^S)\, (1 - \tau_{ss}^E)} - 1 \right) + \beta \mathbb{E}_t \left(\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \right),$$

$$(1 + \pi_t^W) w_{t-1} = w_t (1 + \pi_t^P),$$

while the corresponding Lagrangian becomes:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di)$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W)$$
$$+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_{SS}^L)(1-\tau_{SS}^E)(1-\tau_t^S)} - Z_t \right) L_t$$
$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big)$$
$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W) w_{t-1} - w_t (1+\pi_t^P) \right)$$

We now turn to the computation of the FOCs.

FOC wrt τ_t^S. We derive wrt $\frac{1}{1-\tau_t^S}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{w_t}{(1-\tau_{SS}^L)(1-\tau_{SS}^E)} L_t$$

or

$$\gamma_{P,t} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} w_t = 0,$$

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^L} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$0 = \int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di).$$

FOC wrt L_t.

$$\begin{aligned}\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\ &- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\ &- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\ &+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\ &+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big)\end{aligned}$$

$$\begin{aligned}0 = &- \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) \\ &+ w_t \left(\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di) \right)\end{aligned}$$

Using the FOC wrt w_t:

$$0 = - \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right)$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1+r_{t+1})\hat{\psi}_{i,t+1} \right].$$

E Optimal policies for demand shocks

E.1 Economy 1, with all instruments

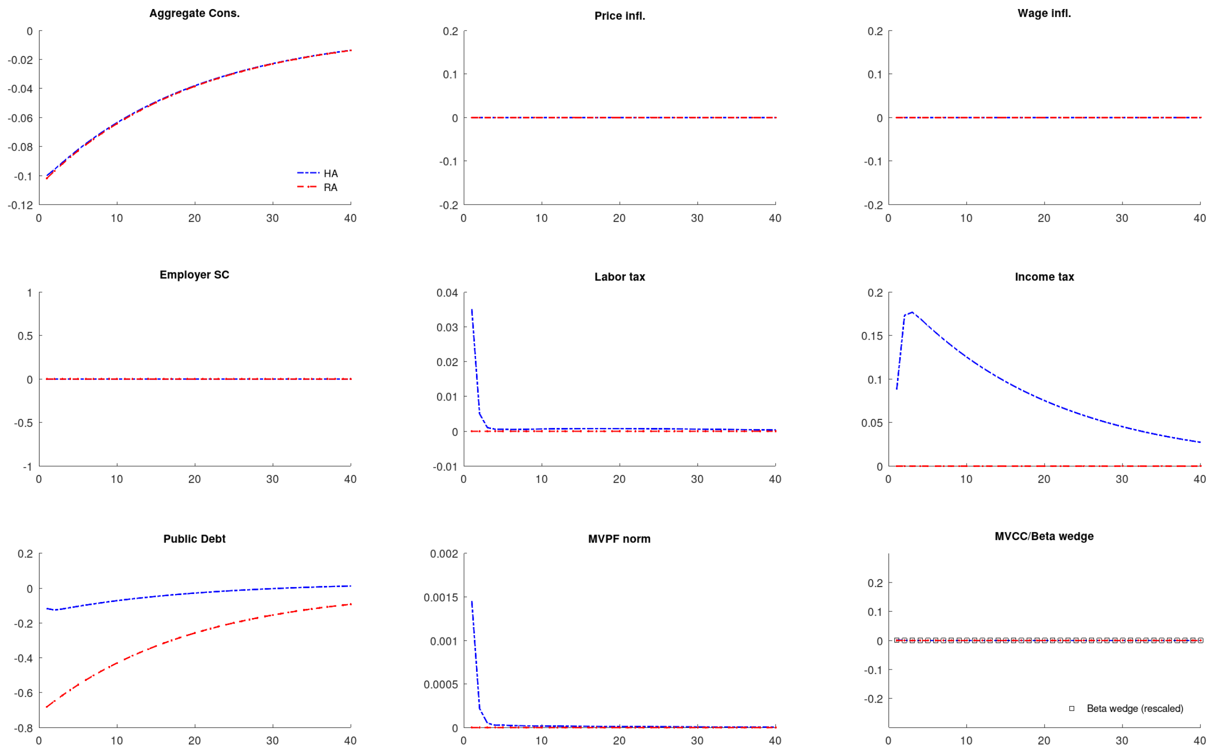

Figure 4: Dynamics of the economy for positive public spending shock for Economy 1 with optimal time-varying $\left\{\tau_t^E, \tau_t^S, \tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

E.2 Economy 3, with time-varying τ_t^L

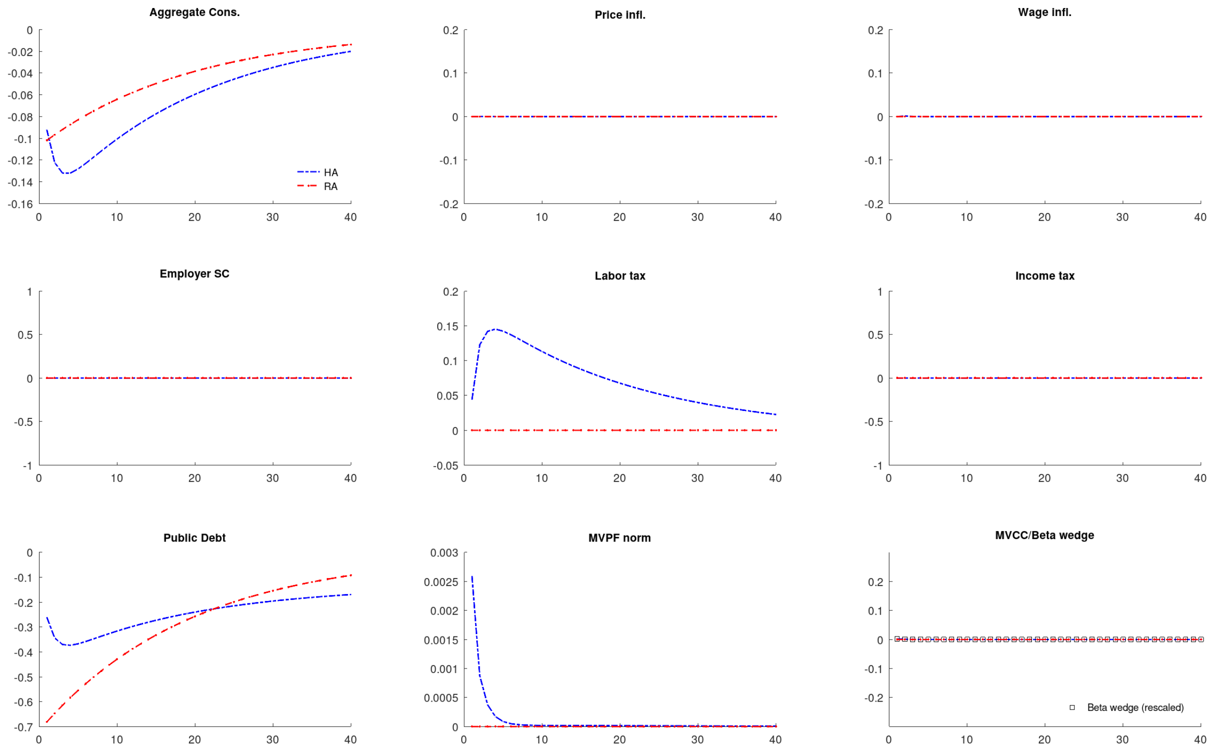

Figure 5: Dynamics of the economy for positive public spending shock for Economy 3 with optimal time-varying $\left\{\tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

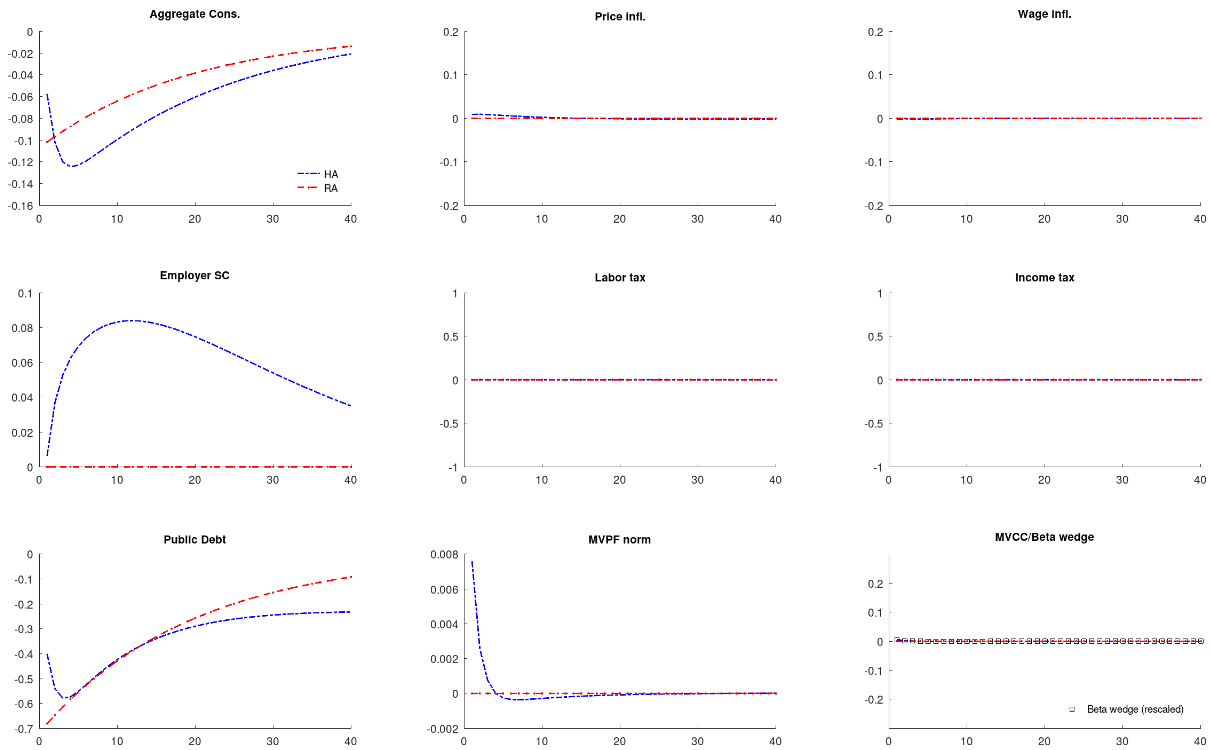

Figure 6: Dynamics of the economy for positive public spending shock for Economy 4 with optimal time-varying $\left\{\tau_t^S\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

E.3 Economy 4, with time-varying τ_t^S

F Optimal policies for supply shocks

F.1 Economy 1, with all instruments

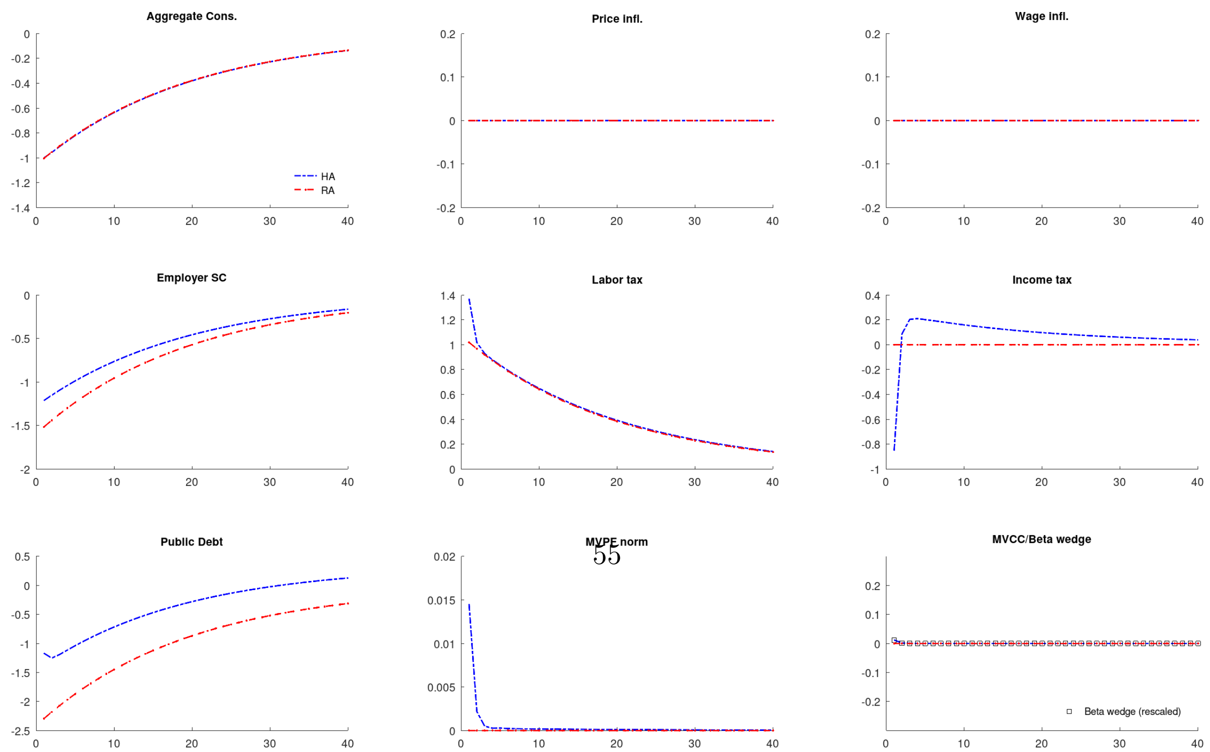

Figure 7: Dynamics of the economy for negative supply shock for Economy 1 with optimal time-varying $\left\{\tau_t^E, \tau_t^S, \tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-

F.2 Economy 3, with time-varying τ_t^L

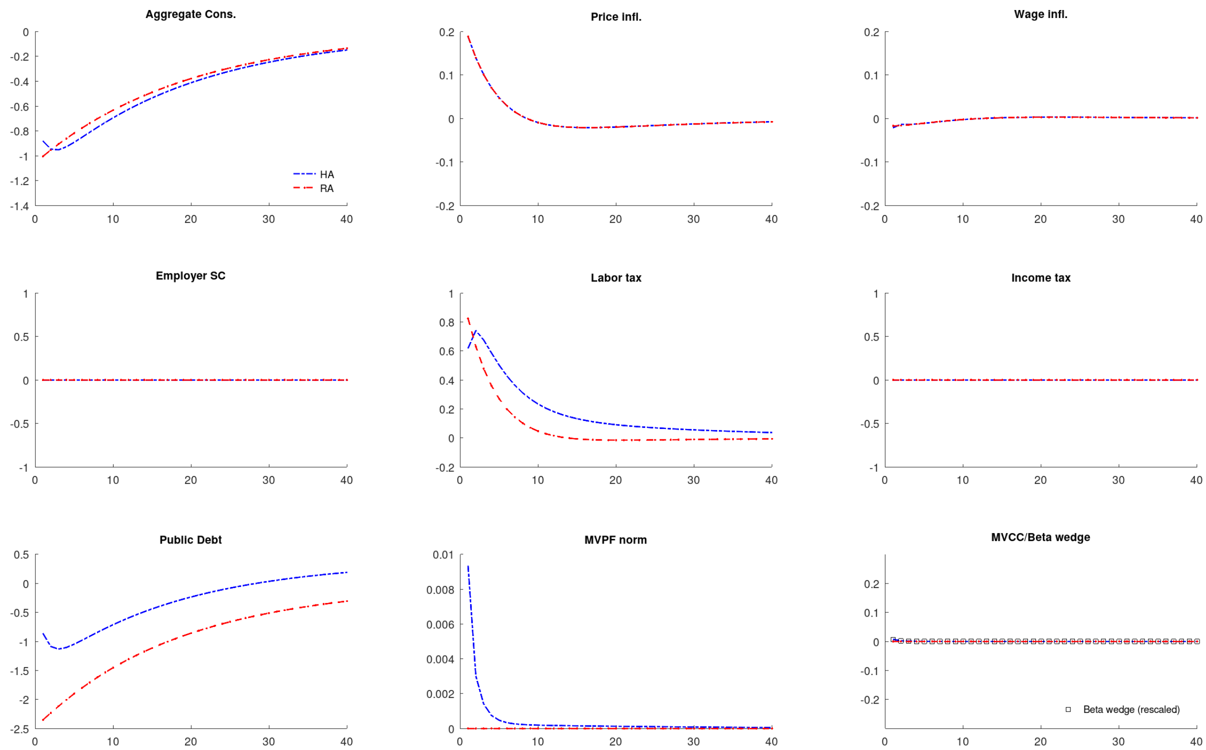

Figure 8: Dynamics of the economy for negative supply shock for Economy 3 with optimal time-varying $\left\{\tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

F.3 Economy 4, with time-varying τ_t^S

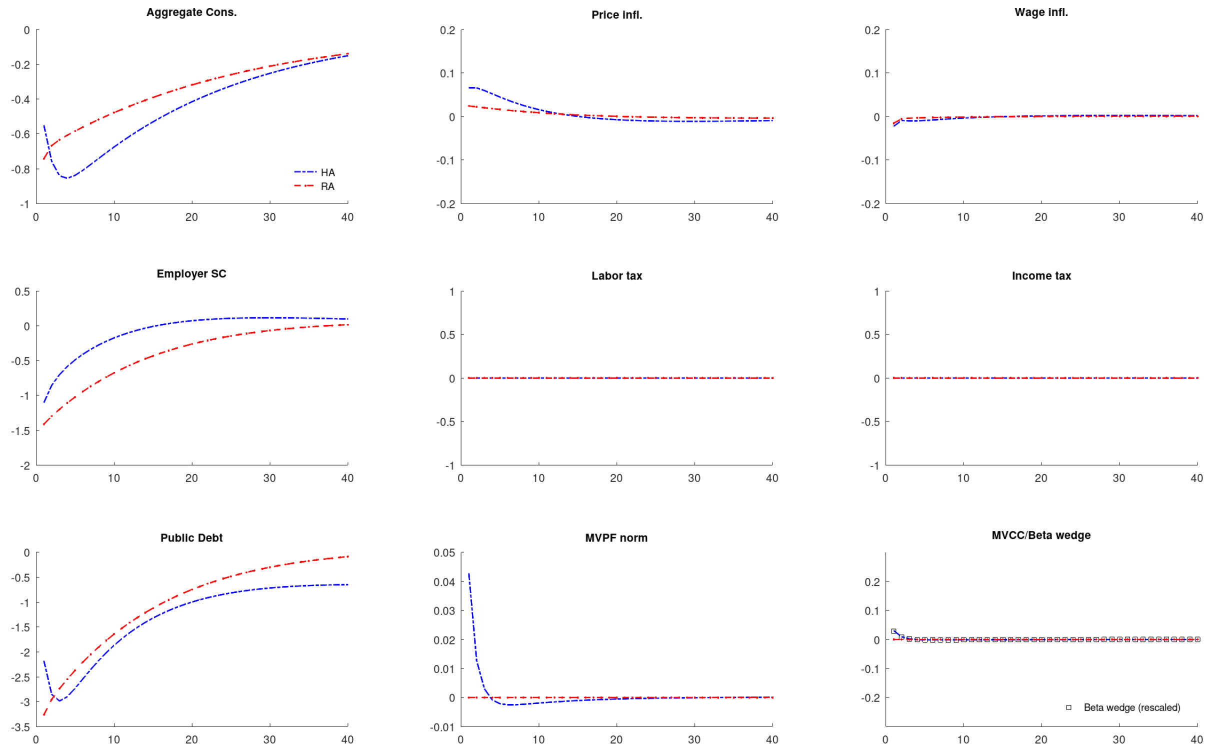

Figure 9: Dynamics of the economy for negative supply shock for Economy 4 with optimal time-varying $\left\{\tau_t^S\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.